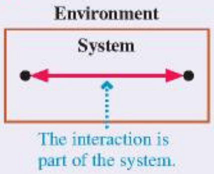
10 Interactions and Potential Energy

These windmills are transforming the kinetic energy of the wind into electric energy.

IN THIS CHAPTER, you will develop a better understanding of energy and its conservation.

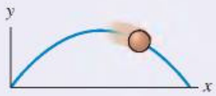
How do interactions affect energy?

We continue our investigation of energy by allowing interactions to be part of the system, rather than external forces. You will learn that interactions can store energy within the system. Further, this interaction energy can be transformed—via the interaction forces—into kinetic energy.



What is potential energy?

Interaction energy is usually called potential energy. There are many kinds of potential energy, each associated with *position*.



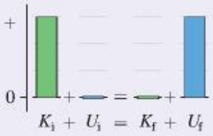
- Gravitational potential energy changes with height.
- Elastic potential energy changes with stretching.

« LOOKING BACK Section 9.1 Energy overview

When is energy conserved?

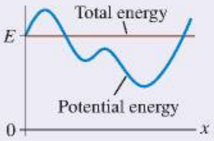
- If a system is isolated, its total energy is conserved.
- If a system both is isolated and has *no dissipative forces*, its mechanical energy, $K + U$, is conserved.

Energy bar charts are a tool for visualizing energy conservation.



What is an energy diagram?

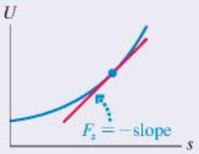
An energy diagram is a graphical representation of how the energy of a particle changes as it moves. Turning points occur where the total energy line crosses the potential-energy curve. And potential-energy minima are points of stable equilibrium.



How is force related to potential energy?

Only certain types of forces, called conservative forces, are associated with a potential energy. For these forces,

- The work done changes the potential energy by $\Delta U = -W$.
- Force is the negative of the slope of the potential-energy curve.



Where are we now in our study of energy?

Energy is a big topic, not one that can be presented in a single chapter. Chapters 9 and 10 are primarily about mechanical energy and the mechanical transfer of energy via work. And we've touched on thermal energy because it's unavoidable in realistic mechanical systems with friction. These are related by the energy principle:

$$\Delta E_{\text{sys}} = \Delta K + \Delta U + \Delta E_{\text{th}} = W_{\text{ext}}$$

Part V of this book, Thermodynamics, will expand our energy ideas to include heat and a deeper understanding of thermal energy. Then we'll add another form of energy—electric energy—in Part VI.

10.1 Potential Energy

If you press a ball against a spring and release it, the ball shoots forward. It certainly seems like the spring had a supply of stored energy that was transferred to the ball. Or imagine tossing the ball straight up. Where does its kinetic energy go as it slows? And from where does it acquire kinetic energy as it falls? There's again a sense that the energy is stored somewhere as the ball rises, then released as the ball falls. But is energy really stored? And if so, where? And how? Answering these questions is key to expanding our understanding of the basic energy model.

Chapter 9 emphasized the importance of the *system* and the *environment.* The system has energy E_{sys}, and forces from the environment—external forces—change the system's energy by doing work on the system. In Chapter 9, we considered only systems of particles, and all forces originated in the environment. But that's not the only way to define the system. What happens if we bring some of the interactions inside the system?

FIGURE 10.1 Two choices of the system and the environment.

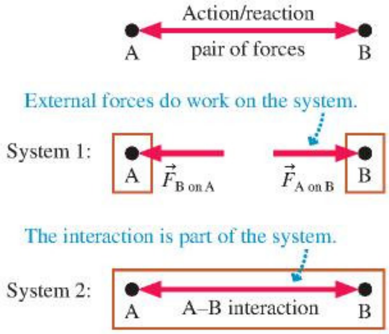


FIGURE 10.1 shows two particle-like objects A and B that interact with each other and nothing else. For example, these might be two objects connected by a spring, two masses exerting gravitational forces on each other, or two charged particles exerting electric forces on each other. Regardless of what the interaction is, this is an action/reaction pair of forces that obeys Newton's third law. There are two ways to define a system.

System 1 has been chosen to consist of only the two particles; the forces are external forces. This is exactly the analysis we did in Chapter 9, so we know that the energy principle for system 1 is

$$\Delta E_{\text{sys 1}} = \Delta K_{\text{tot}} = W_{\text{ext}} = W_{\text{A}} + W_{\text{B}} \tag{10.1}$$

where K_{tot} is the combined kinetic energies of A and B. Work W_{A} is the work done on A by force $\vec{F}_{\text{B on A}}$, and similarly W_{B} is the work done on B by force $\vec{F}_{\text{A on B}}$. The work of these two forces changes the system's kinetic energy.

Now consider the same two particles but with a different choice of system, system 2, where we've included the interaction within the system. It's important to recognize that a system is not a physical thing. It's an analysis tool that we can define however we wish, and our choice doesn't change the behavior of physical objects. Objects A and B are oblivious to our choice of system, so ΔK_{tot} for system 2 is exactly the same as for system 1. But W_{ext} *has* changed. System 2 has no external forces to transfer energy to or from the system, so $W_{\text{ext}} = 0$. Consequently, the energy principle for system 2 is

$$\Delta E_{\text{sys 2}} = W_{\text{ext}} = 0 \tag{10.2}$$

Now the fact that $\Delta E_{\text{sys 1}} \neq \Delta E_{\text{sys 2}}$ is not an issue; after all, they are different systems. But we know that system 2 has a changing kinetic energy, so how can $\Delta E_{\text{sys 2}} = 0$?

Because system 2 has an interaction inside the system that system 1 lacks, let's postulate that system 2 has an additional form of energy associated with the interaction. That is, system 1 has $E_{\text{sys 1}} = K_{\text{tot}}$, because particles have only kinetic energy, but system 2 has $E_{\text{sys 2}} = K_{\text{tot}} + U$, where U, called **potential energy,** is the energy of the interaction.

If this is true, we can combine $\Delta E_{\text{sys 2}} = 0$, from Equation 10.2, with our knowledge of ΔK_{tot} from Equation 10.1 to write

$$\Delta E_{\text{sys 2}} = \Delta K_{\text{tot}} + \Delta U = (W_{\text{A}} + W_{\text{B}}) + \Delta U = 0 \tag{10.3}$$

That is, system 2 can have $\Delta E_{\text{sys}} = 0$ if it has a potential energy that changes by

$$\Delta U = -(W_{\text{A}} + W_{\text{B}}) = -W_{\text{int}} \tag{10.4}$$

where W_{int} is the total work done *inside the system* by the interaction forces.

Equation 10.3 tells us that the system's kinetic energy can increase ($\Delta K > 0$) if its potential energy decreases ($\Delta U < 0$) by the same amount. In effect, **the interaction stores energy inside the system** with the *potential* to be converted to kinetic energy (or, in other situations, thermal energy)—hence the name *potential energy.* This idea will become more concrete as we start looking at specific examples. And, since we *postulated* the existence of an energy associated with interactions, we'll need to investigate the types of interactions for which this is true.

> **NOTE** Kinetic energy is the energy of an object. In contrast, potential energy is the energy of an interaction. You can say "The ball has kinetic energy" but not "The ball has potential energy." We'll look at the best way to describe potential energy when we get to specific examples.

Systems Matter

When solving a problem, *you* get to define the system. But your choice has consequences! E_{sys} is the energy *of* the system, so a different system will have a different energy. Similarly, W_{ext} is the work done on the system by forces originating in the environment, and that will depend on the boundary between the system and the environment.

In Figure 10.1, system 1 is a restricted system of just the particles, so system 1 has only kinetic energy. All the interaction forces are external forces that do work. Thus system 1 obeys

$$\Delta E_{\text{sys}} = \Delta K_{\text{tot}} = W_{\text{A}} + W_{\text{B}}$$

System 2 includes the interaction, so system 2 has both kinetic and potential energy. But the choice of the system boundary means that no work is done by external forces. So for system 2,

$$\Delta E_{\text{sys}} = \Delta K_{\text{tot}} + \Delta U = 0$$

Both mathematical statements are true because they refer to different systems. Notice that, for system 2, kinetic energy can be transformed into potential energy, or vice versa, but **the total energy of the system does not change.** This is our first glimpse of the idea of *conservation of energy*.

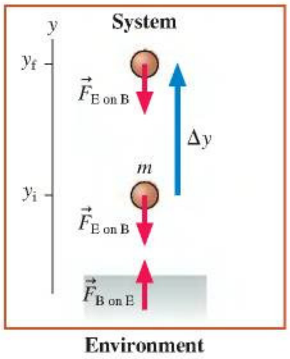
The point to remember is that **any choice of system is acceptable, but you can't mix and match.** You can define the system so that you have to calculate work, or you can define the system where you use potential energy, but using both work *and* potential energy is incorrect because it double counts the contribution of the interaction. Thus the most critical step in an energy analysis is to clearly define the system you're working with.

10.2 Gravitational Potential Energy

We'll start our exploration of potential energy with **gravitational potential energy,** the interaction energy associated with the gravitational interaction between two masses. The symbol for gravitational potential energy is U_{G}. We'll restrict ourselves to the "flat-earth approximation" $F_{\text{G}} = -mg\hat{\jmath}$. The gravitational potential energy of two astronomical bodies will be taken up in Chapter 13.

FIGURE 10.2 shows a ball of mass m moving upward from an initial vertical position y_{i} to a final vertical position y_{f}. The earth exerts force $\vec{F}_{\text{E on B}}$ on the ball and, by Newton's third law, the ball exerts an equal-but-opposite force $\vec{F}_{\text{B on E}}$ on the earth.

FIGURE 10.2 The ball + earth system has a gravitational potential energy.



We could define the system to consist of only the ball, in which case the force of gravity is an external force that does work on the ball, changing its kinetic energy. We did exactly this in Chapter 9. Now let's define the system to be ball + earth. This brings the interaction inside the system, so (ignoring any gravitational forces from distant astronomical bodies) there's no external work. Instead, we have an energy of interaction—the gravitational potential energy—described by Equation 10.4:

$$\Delta U_{\text{G}} = -(W_{\text{B}} + W_{\text{E}}) \tag{10.5}$$

where W_{B} is the work gravity does on the ball and W_{E} is the work gravity does on the earth. The latter, practically speaking, is zero. $\vec{F}_{\text{E on B}}$ and $\vec{F}_{\text{B on E}}$ have equal magnitudes, by Newton's third law, but the earth's displacement is completely insignificant compared to the ball's displacement. Because work is a product of force and displacement, the work done on the earth is essentially zero and we can write

$$\Delta U_G = -W_B \tag{10.6}$$

You learned in Chapter 9 to compute the work of gravity on the ball: $W_B = (F_G)_y \Delta y = -mg\,\Delta y$. So if the ball changes its vertical position by Δy, the gravitational potential energy changes by

$$\Delta U_G = -W_B = mg\,\Delta y \tag{10.7}$$

Notice that increasing the ball's height ($\Delta y > 0$) increases the gravitational potential energy ($\Delta U_G > 0$), as we would expect.

Our energy analysis has given us an expression for ΔU_G, the *change* in potential energy, but not an expression for U_G itself. If we write out what the Δ in Equation 10.7 means—final value minus initial value—we have

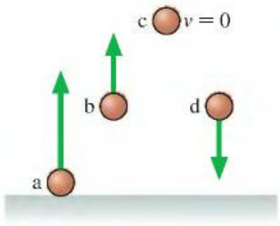
$$U_{Gf} - U_{Gi} = mgy_f - mgy_i \tag{10.8}$$

Consequently, we define the gravitational potential energy to be

$$U_G = mgy \qquad \text{(gravitational potential energy)} \tag{10.9}$$

Notice that **gravitational potential energy is an energy of position.** It depends on the object's position but not on its speed. You should convince yourself that the units of mass times acceleration times position are joules, the unit of energy.

STOP TO THINK 10.1 Rank in order, from largest to smallest, the gravitational potential energies of balls a to d.



EXAMPLE 10.1 Launching a pebble

Rafael uses a slingshot to shoot a 25 g pebble straight up at 17 m/s. How high does the pebble go?

MODEL Let the system consist of both the earth and the pebble, which we model as a particle. Assume that air resistance is negligible. There are no external forces to do work, but the system does have gravitational potential energy.

VISUALIZE **FIGURE 10.3** is a before-and-after pictorial representation. The before-and-after representation will continue to be our primary visualization tool.

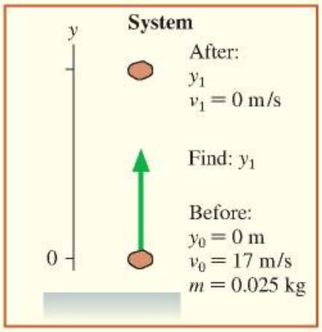
SOLVE The energy principle for the pebble + earth system is

$$\Delta E_{sys} = \Delta K + \Delta U_G = W_{ext} = 0$$

That is, the system energy does not change at all. Instead, kinetic energy is transformed into potential energy without loss inside the system. In principle, the kinetic energy is that of the ball plus the kinetic energy of the earth. But as we just noted, the enormous mass difference means that the earth is effectively at rest while the pebble does all the moving, so the only kinetic energy we need to consider is that of the pebble. Thus we have

$$0 = \Delta K + \Delta U_G = \left(\tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2\right) + (mgy_1 - mgy_0)$$

FIGURE 10.3 Pictorial representation of the pebble + earth system.



We know that $v_1 = 0$ m/s, and we chose a coordinate system in which $y_0 = 0$ m, so we're left with

$$y_1 = \frac{v_0^2}{2g} = \frac{(17\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 15\text{ m}$$

The answer did not depend on the pebble's mass, which is not surprising after our earlier practice with free-fall problems.

ASSESS A height of 15 m ≈ 45 ft seems reasonable for a slingshot.

The Zero of Potential Energy

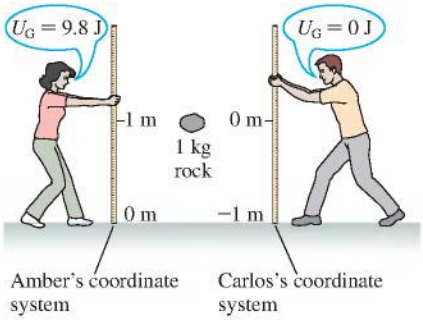
Our expression for the gravitational potential energy $U_G = mgy$ seems straightforward. But you might notice, on further reflection, that the value of U_G depends on where you choose to put the origin of your coordinate system. Consider **FIGURE 10.4**, where Amber and Carlos are attempting to determine the potential energy when a 1 kg rock is 1 m above the ground. Amber chooses to put the origin of her coordinate system on the ground, measures $y_{\text{rock}} = 1\text{ m}$, and quickly computes $U_G = mgy = 9.8\text{ J}$. Carlos, on the other hand, read Chapter 1 very carefully and recalls that it is entirely up to him where to locate the origin of his coordinate system. So he places his origin next to the rock, measures $y_{\text{rock}} = 0\text{ m}$, and declares that $U_G = mgy = 0\text{ J}$!

How can the potential energy have two different values? The source of this apparent difficulty comes from our interpretation of Equation 10.7. Our energy analysis found that the potential energy *changes* by $\Delta U_G = mg(y_f - y_i)$. Our claim that $U_G = mgy$ is consistent with this finding, but so also would be a claim that $U_G = mgy + C$, where C is any constant.

In other words, potential energy does not have a uniquely defined value. Adding or subtracting the same constant from all potential energies in a problem has no physical consequences because our analysis uses only ΔU_G, the *change* in the potential energy, never the actual value of U_G. In practice, we work with potential energies by setting a *reference point* or *reference level* where $U_G = 0$. This is the **zero of potential energy.** Where you place the reference point is entirely up to you; it makes no difference as long as every potential energy in the problem uses the same reference point. For gravitational potential energy, we choose the reference level by placing the origin of the y-axis at that point. Where $y = 0$, $U_G = 0$. In Figure 10.4, Amber has placed her zero of potential energy at the ground, whereas Carlos has set a reference level 1 m above the ground. Either is perfectly acceptable as long as Amber and Carlos use their reference levels consistently.

But what happens when the rock falls? When it gets to the ground, Amber measures $y = 0\text{ m}$ and computes $U_G = 0\text{ J}$. No problem. But Carlos measures $y = -1\text{ m}$ and thus computes $U_G = -9.8\text{ J}$. A negative potential energy may seem surprising, but it's not wrong; it simply means that the potential energy is less than at the reference point. The potential energy with the rock on the ground is certainly less than when the rock was 1 m above the ground, so for Carlos—with an elevated reference level—the potential energy is negative. The important point is that both Amber and Carlos agree that the gravitational potential energy *changes* by $\Delta U_G = -9.8\text{ J}$ as the rock falls.

FIGURE 10.4 Amber and Carlos use different coordinate systems to determine the gravitational potential energy.



Energy Bar Charts

If an object of mass m interacts with the earth (or other astronomical body) and there are no other forces, the energy principle for the object + earth system, Equation 10.3, is

$$\Delta K + \Delta U_G = (K_f - K_i) + (U_{Gf} - U_{Gi}) = 0 \tag{10.10}$$

We can rewrite this as

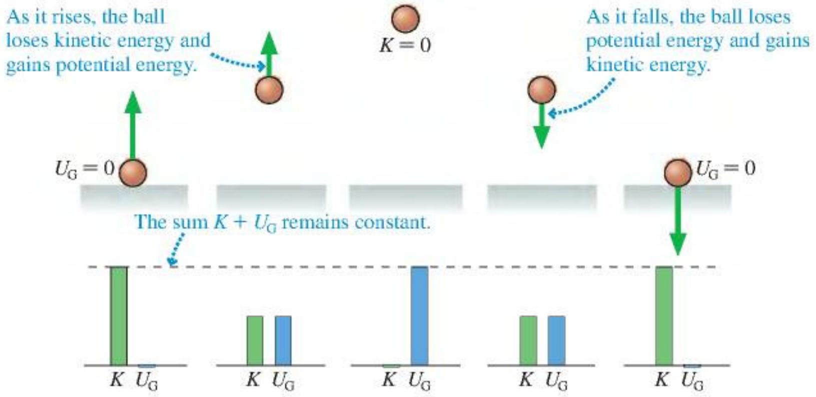
$$K_i + U_{Gi} = K_f + U_{Gf} \tag{10.11}$$

The quantity $E_{\text{mech}} = K_{\text{tot}} + U_{\text{int}}$, the total macroscopic kinetic and potential energy, is called the **mechanical energy** of the system. Equation 10.11 is telling us that—in this situation—**the mechanical energy does not change** as the object undergoes vertical motion. Whatever initial mechanical energy the system had before the vertical motion, it has exactly the same mechanical energy after the motion. Kinetic energy may be transformed into potential energy during the motion, or vice versa, but their sum remains unchanged.

A quantity that is unchanged during an interaction is said to be *conserved*, and Equation 10.11 is our first statement of the *law of conservation of energy*. Now this is a highly restricted situation: only the gravitational force, no other forces, and only vertical motion. We'll explore energy conservation thoroughly later in this chapter and see to what extent these restrictions can be lifted, but we're already beginning to see the power of thinking about mechanical systems in terms of energy.

Equation 10.11, which is really just energy accounting, can be represented graphically with an **energy bar chart.** For example, FIGURE 10.5 is a bar chart showing how energy is transformed when a ball is tossed straight up. Kinetic energy is gradually transformed into potential energy as the ball rises, then potential energy is transformed into kinetic energy as it falls, but **the combined height of the bars does not change.** That is, the mechanical energy of the ball + earth system is conserved.

FIGURE 10.5 Energy bar charts for a ball tossed into the air.



NOTE Most bar charts have no numbers. Their purpose is to think about the relative changes—what's increasing, what's decreasing, and what remains constant; there's no significance to how tall a bar is.

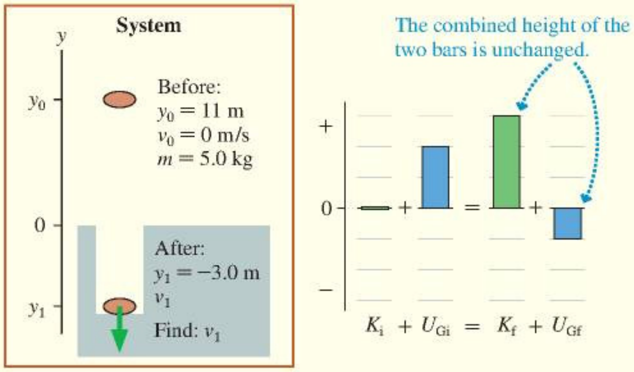
EXAMPLE 10.2 Dropping a watermelon

A 5.0 kg watermelon is dropped from a third-story balcony, 11 m above the street. Unfortunately, the water department forgot to replace the cover on a manhole, and the watermelon falls into a 3.0-m-deep hole. How fast is the watermelon going when it hits bottom?

MODEL Let the system consist of both the earth and the watermelon, which we model as a particle. Assume that air resistance is negligible. There are no external forces, and the motion is vertical, so the system's mechanical energy is conserved.

VISUALIZE FIGURE 10.6 shows both a before-and-after pictorial representation and an energy bar chart. Initially the system has gravitational potential energy but no kinetic energy. Potential energy is transformed into kinetic energy as the watermelon falls. Our choice of the y-axis origin has placed the zero of potential energy at ground level, so the potential energy is negative when the watermelon reaches the bottom of the hole. Even so, the combined height of the two bars has not changed.

FIGURE 10.6 Pictorial representation and energy bar chart of the watermelon + earth system.



SOLVE The energy principle for the watermelon + earth system, written as a conservation statement, is

$$K_i + U_{Gi} = 0 + mgy_0 = K_f + U_{Gf} = \tfrac{1}{2}mv_1^2 + mgy_1$$

Solving for the impact speed, we find

$$\begin{aligned} v_1 &= \sqrt{2g(y_0 - y_1)} \\ &= \sqrt{2(9.80\text{ m/s}^2)(11.0\text{ m} - (-3.0\text{ m}))} \\ &= 17\text{ m/s} \end{aligned}$$

ASSESS A speed of 17 m/s ≈ 35 mph seems reasonable for the watermelon after falling ≈ 4 stories. In thinking about this problem, you might be concerned that, once below ground level, potential energy continues being transformed into kinetic energy even though the potential energy is "less than none." Keep in mind that the actual value of U is not relevant because we can place the zero of potential energy anywhere we wish, so a negative potential energy is just a number with no implication that it's "less than none." There's no "storehouse" of potential energy that might run dry. As long as the interaction acts, potential energy can continue being transformed into kinetic energy.

Digging Deeper into Gravitational Potential Energy

The concept of gravitational potential energy would be of little interest or use if it applied only to vertical free fall. Let's begin to expand the idea. **FIGURE 10.7** shows a particle of mass m moving at an angle while acted on by gravity. How much work does gravity do?

Gravity is a constant force. In Chapter 9 you learned that, in general, the work done by a constant force is $W = \vec{F} \cdot \Delta\vec{r}$. If we write both $\vec{F}_G$ and $\Delta\vec{r}$ in terms of components, and use the Chapter 9 result for calculating the dot product with components, we find that the work done by gravity is

$$\begin{aligned} W_{\text{by grav}} &= \vec{F}_G \cdot \Delta\vec{r} = (F_G)_x(\Delta r_x) + (F_G)_y(\Delta r_y) = 0 + (-mg)(\Delta y) \\ &= -mg\,\Delta y \end{aligned} \tag{10.12}$$

Because $\vec{F}_G$ has no x-component, the work depends only on the vertical displacement Δy.

Consequently, **the change in gravitational potential energy depends only on an object's vertical displacement.** This is true not only for motion along a straight line, as in Figure 10.7, but also for motion along a *curved* trajectory because a curve can be represented as the limit of a very large number of very short straight-line segments.

For example, **FIGURE 10.8** shows an object sliding down a curved, frictionless surface. The change in gravitational potential energy of the object + earth system depends only on Δy, the distance the object descends, *not* on the shape of the curve. But now there's an additional force—the normal force of the surface. Does this force affect the system's energy? No! The normal force is always perpendicular to the box's instantaneous displacement, and you learned in Chapter 9 that forces perpendicular to the displacement do no work. **Forces always perpendicular to the motion do not affect the system's energy.** They can be ignored during an energy analysis.

FIGURE 10.7 Gravity does work on a particle moving at an angle.

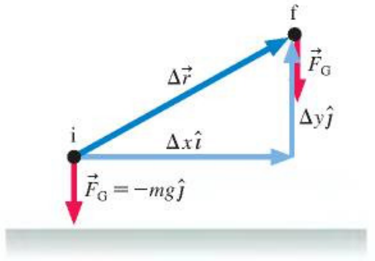
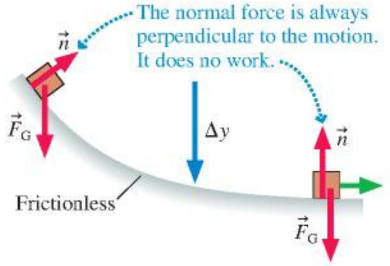
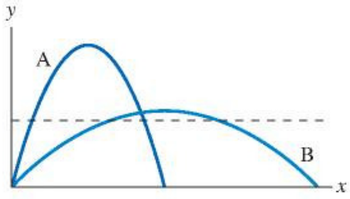


FIGURE 10.8 For motion on any frictionless surface, only the vertical displacement Δy affects the energy.


The normal force is always perpendicular to the motion. It does no work.

STOP TO THINK 10.2 Two identical projectiles are fired with the same speed but at different angles. Neglect air resistance. At the elevation shown as a dashed line,



a. The speed of A is greater than the speed of B.
b. The speed of A is the same as the speed of B.
c. The speed of A is less than the speed of B.

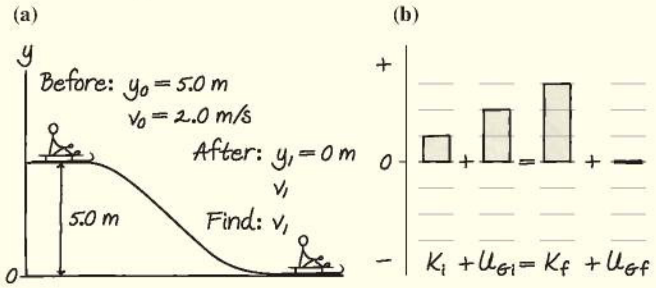
EXAMPLE 10.3 The speed of a sled

Christine runs forward with her sled at 2.0 m/s. She hops onto the sled at the top of a 5.0-m-high, very slippery slope. What is her speed at the bottom?

MODEL Let the system consist of the earth and the sled, which we model as a particle. Because the slope is "very slippery," we'll assume that friction is negligible. The slope exerts a normal force on the sled, but it is always perpendicular to the motion and does not affect the energy.

VISUALIZE **FIGURE 10.9a** shows a before-and-after pictorial representation. We are not told the angle of the slope, or even if it is a straight slope, but the change in potential energy depends only on the vertical distance Christine descends and *not* on the shape of the hill. **FIGURE 10.9b** is an energy bar chart in which we see an initial

FIGURE 10.9 Pictorial representation and energy bar chart of the sled + earth system.


(a) (b)

Continued

kinetic *and* potential energy being transformed into entirely kinetic energy as Christine goes down the slope.

SOLVE The energy analysis is just like in Example 10.2; the fact that the object is moving on a curved surface hasn't changed anything. The energy principle, written as a conservation statement, is

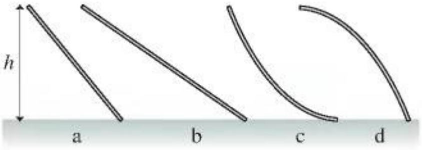
$$\begin{aligned} K_i + U_{Gi} &= \tfrac{1}{2}mv_0^2 + mgy_0 \\ &= K_f + U_{Gf} = \tfrac{1}{2}mv_1^2 + 0 \end{aligned}$$

Her speed at the bottom is

$$\begin{aligned} v_1 &= \sqrt{v_0^2 + 2gy_0} \\ &= \sqrt{(2.0\ \text{m/s})^2 + 2(9.80\ \text{m/s}^2)(5.0\ \text{m})} \\ &= 10\ \text{m/s} \end{aligned}$$

ASSESS 10 m/s ≈ 20 mph is fast but believable for a 5 m ≈ 15 ft descent.

STOP TO THINK 10.3 A small child slides down the four frictionless slides a–d. Each has the same height. Rank in order, from largest to smallest, her speeds v_a to v_d at the bottom.



Motion with Gravity and Friction

What if there's friction? You learned in « Section 9.5 that friction increases the thermal energy of the system—defined to include *both* objects—by $\Delta E_{th} = f_k \Delta s$. For a system with both gravitational potential energy and friction, the energy principle becomes

$$\Delta K + \Delta U_G + \Delta E_{th} = 0 \tag{10.13}$$

or, equivalently,

$$K_i + U_{Gi} = K_f + U_{Gf} + \Delta E_{th} \tag{10.14}$$

Mechanical energy $K + U_G$ is *not* conserved if there is friction. Because $\Delta E_{th} > 0$ (friction always makes surfaces hotter, never cooler), the final mechanical energy is less than the initial mechanical energy. That is, some fraction of the initial kinetic and potential energy is transformed into thermal energy during the motion. Friction causes objects to slow down, and motion ceases when all the mechanical energy has been transformed into thermal energy. Mechanical energy is conserved only when there are no dissipative forces and thus $\Delta E_{th} = 0$.

NOTE We can write the energy principle in terms of initial and final values of the kinetic energy and the potential energy, but *not* the thermal energy. Objects always have thermal energy—the atoms are constantly in motion—but we have no way to know how much. All we can calculate is the *change* in thermal energy.

Although mechanical energy is not conserved, *the system's energy is.* Equation 10.13 tells us that the sum of kinetic, potential, and thermal energy—the energy of the system—does not change as the object moves on a surface with friction. The initial mechanical energy does not disappear; it's merely transformed into an equal amount of thermal energy.

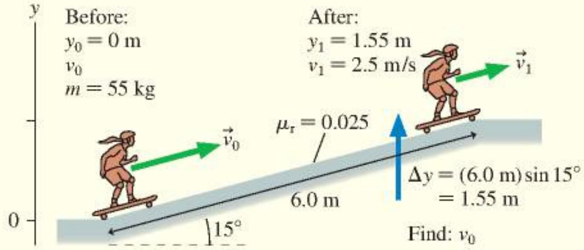
EXAMPLE 10.4 Skateboarding up a ramp

During the skateboard finals, Isabella encounters a 6.0-m-long, 15° upward ramp. Isabella's mass, including the skateboard, is 55 kg, and the coefficient of rolling friction between her wheels and the ramp is 0.025. With what speed must she start up the ramp to reach the top at 2.5 m/s? What percentage of her mechanical energy is "lost" to friction?

MODEL Let the system consist of the earth (including the ramp) and Isabella on the skateboard.

VISUALIZE **FIGURE 10.10** shows a before-and-after pictorial representation.

FIGURE 10.10 Pictorial representation of Isabella on the ramp.



SOLVE Isabella's kinetic energy is transformed into potential energy as she gains height, but some of her kinetic energy is also transformed into increased thermal energy of her wheels and the ramp because of rolling friction. The energy principle including friction is

$$
\begin{aligned}
K_i + U_{Gi} &= \tfrac{1}{2}mv_0^2 + 0 \\
&= K_f + U_{Gf} + \Delta E_{th} = \tfrac{1}{2}mv_1^2 + mgy_1 + f_r \Delta s
\end{aligned}
$$

where we've used rolling friction f_r rather than the kinetic friction of sliding. Rolling friction is $f_r = \mu_r n$, and recall—from Chapter 6—that the normal force of an object on a slope is $n = mg\cos\theta$. (Draw a free-body diagram if you're not sure.) Thus

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv_1^2 + mgy_1 + \mu_r mg\,\Delta s\cos\theta$$

The mass cancels. Solving for Isabella's speed at the bottom of the ramp, we find

$$v_0 = \sqrt{v_1^2 + 2gy_1 + 2\mu_r g\,\Delta s\cos\theta} = 6.3 \text{ m/s}$$

Isabella's initial mechanical energy is entirely kinetic energy: $K_0 = \frac{1}{2}mv_0^2 = 1090$ J. The thermal energy of the ramp and her wheels increases by $\Delta E_{th} = \mu_r mg\,\Delta s\cos\theta = 78$ J. Thus the percentage of mechanical energy transformed into thermal energy as Isabella ascends the ramp is

$$\frac{78 \text{ J}}{1090 \text{ J}} \times 100 = 7.2\%$$

This energy is not truly lost—it's still in the system—but it's no longer available for motion.

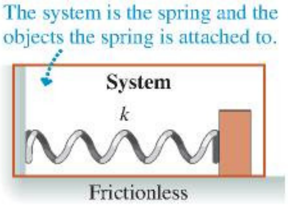
ASSESS The ramp is 1.55 m ≈ 5 ft high. Starting up the ramp at 6.3 m/s ≈ 12 mph in order to reach the top at 2.5 m/s ≈ 5 mph seems reasonable.

STOP TO THINK 10.4 A skier glides down a gentle slope at constant speed. What energy transformation is taking place?

a. $U_G \rightarrow K$
b. $U_G \rightarrow K + E_{th}$
c. $U_G \rightarrow E_{th}$
d. $K \rightarrow E_{th}$
e. No energy transformation is occurring.

10.3 Elastic Potential Energy

FIGURE 10.11 The block + spring + wall system has an elastic potential energy.


The system is the spring and the objects the spring is attached to.

Much of what you've just learned about gravitational potential energy carries over to the *elastic potential energy* of a spring. **FIGURE 10.11** shows a spring exerting a force on a block while the block moves on a frictionless, horizontal surface. In Chapter 9, we analyzed this problem by defining the system to consist of only the block, and we calculated the work of the spring on the block. Now let's define the system to be block + spring + wall. That is, the system is the spring and the objects connected by the spring. The surface and the earth exert forces on the block—the normal force and gravity—but those forces are always perpendicular to the displacement and do not transfer any energy to the system.

We'll assume the spring to be massless, so it has no kinetic energy. Instead, the spring is the *interaction* between the block and the wall. Because the interaction is inside the system, it has an interaction energy, the **elastic potential energy,** given by

$$\Delta U_{Sp} = -(W_B + W_W) \tag{10.15}$$

where W_B is the work the spring does on the block and W_W is the work done on the wall. But the wall is rigid and has no displacement, so $W_W = 0$ and thus $\Delta U_{Sp} = -W_B$.

We calculated the work done by an ideal spring—one that produces a linear restoring force for all displacement—in « Section 9.4. If the block moves from an initial position

s_i, where the spring's displacement is $\Delta s_i = s_i - s_{eq}$, to a final position s_f with displacement $\Delta s_f = s_f - s_{eq}$, the spring does work

$$W_B = -\left(\tfrac{1}{2}k(\Delta s_f)^2 - \tfrac{1}{2}k(\Delta s_i)^2\right) \tag{10.16}$$

With the minus sign of Equation 10.15, we have

$$\Delta U_{Sp} = U_f - U_i = -W_B = \tfrac{1}{2}k(\Delta s_f)^2 - \tfrac{1}{2}k(\Delta s_i)^2 \tag{10.17}$$

Thus the elastic potential energy is

$$U_{Sp} = \tfrac{1}{2}k(\Delta s)^2 \qquad \text{(elastic potential energy)} \tag{10.18}$$

where Δs is the displacement of the spring from its equilibrium length. Elastic potential energy, like gravitational potential energy, is an *energy of position*. It depends on where the block is, not on how fast the block is moving. Although we derived Equation 10.18 for a spring, it applies to *any* linear restoring force if k is the appropriate "spring constant" for that force.

The energy principle for a system with elastic potential energy and no external interactions is either $\Delta E_{sys} = \Delta K + \Delta U_{Sp} = 0$ or, recognizing that mechanical energy is again conserved,

$$K_i + U_{Sp\,i} = K_f + U_{Sp\,f} \tag{10.19}$$

EXAMPLE 10.5 An air-track glider compresses a spring

In a laboratory experiment, your instructor challenges you to figure out how fast a 500 g air-track glider is traveling when it collides with a horizontal spring attached to the end of the track. He pushes the glider, and you notice that the spring compresses 2.7 cm before the glider rebounds. After discussing the situation with your lab partners, you decide to hang the spring on a hook and suspend the glider from the bottom end of the spring. This stretches the spring by 3.5 cm. Based on your measurements, how fast was the glider moving?

MODEL Let the system consist of the track, the spring, and the glider. The spring is inside the system, so the elastic interaction will be treated as a potential energy. Gravity and the normal force of the track on the glider are perpendicular to the glider's displacement, so they do no work and do not enter into an energy analysis. An air track is essentially frictionless, and there are no other external forces.

VISUALIZE FIGURE 10.12 shows a before-and-after pictorial representation of the collision, an energy bar chart, and a free-body diagram of the suspended glider.

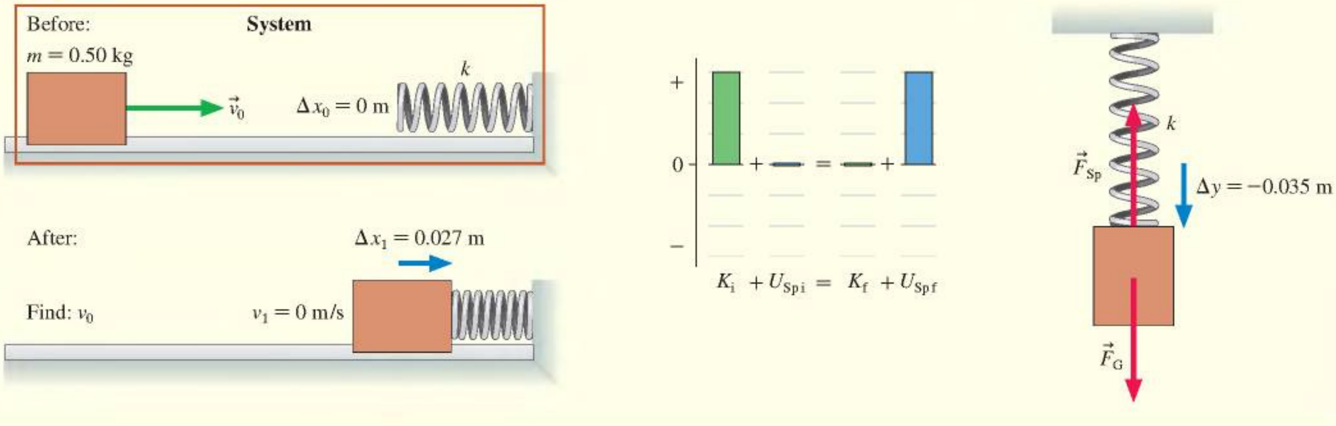
SOLVE The glider's kinetic energy is gradually transformed into elastic potential energy as it compresses the spring. The point of maximum compression—After in Figure 10.12—is a turning point in the motion. The velocity is instantaneously zero, the glider's kinetic energy is zero, and thus—as the bar chart shows—all the energy has been transformed into potential energy. The spring will expand and cause the glider to rebound, but that's not part of this problem. The energy principle with elastic potential energy, in conservation form, is

$$K_i + U_{Sp\,i} = \tfrac{1}{2}mv_0^2 + 0 = K_f + U_{Sp\,f} = 0 + \tfrac{1}{2}k(\Delta x_1)^2$$

where we utilized our knowledge that the initial elastic potential energy and the final kinetic energy, at the turning point, are zero. Solving for the glider's initial speed, we find

$$v_0 = \sqrt{\frac{k}{m}}\,\Delta x_1$$

FIGURE 10.12 Pictorial representation of the experiment.



It was at this point that you and your lab partners realized you needed to determine the spring constant k. One way to do so is to measure the stretch caused by a suspended mass. The hanging glider is in equilibrium with no net force, and the free-body diagram shows that the upward spring force exactly balances the downward gravitational force. From Hooke's law, the *magnitude* of the spring force is $F_{\text{Sp}} = k|\Delta y|$. Thus Newton's first law for the suspended glider is

$$F_{\text{Sp}} = k|\Delta y| = F_{\text{G}} = mg$$

from which the spring's spring constant is

$$k = \frac{mg}{|\Delta y|} = \frac{(0.50\text{ kg})(9.80\text{ m/s}^2)}{0.035\text{ m}} = 140\text{ N/m}$$

Knowing k, you can now find that the glider's speed was

$$v_0 = \sqrt{\frac{k}{m}}\,\Delta x_1 = \sqrt{\frac{140\text{ N/m}}{0.50\text{ kg}}}\,(0.027\text{ m}) = 0.45\text{ m/s}$$

ASSESS A speed of ≈ 0.5 m/s is typical for gliders on an air track.

Including Gravity

Now that we see how the basic energy model works, it's easy to extend it to new situations. If a problem has both a spring *and* a vertical displacement, we define the system so that both the gravitational interaction and the elastic interaction are inside the system. Then we have both elastic *and* gravitational potential energy. That is,

$$U = U_{\text{G}} + U_{\text{Sp}} \tag{10.20}$$

You have to be careful with the energy accounting because there are more ways that energy can be transformed, but nothing fundamental has changed by having two potential energies rather than one.

And we know how to include the increased thermal energy if there's friction. Thus for a system that has gravitational interactions, elastic interactions, and friction, but no external forces that do work, the energy principle is

$$\Delta E_{\text{sys}} = \Delta K + \Delta U_{\text{G}} + \Delta U_{\text{Sp}} + \Delta E_{\text{th}} = 0 \tag{10.21}$$

or, in conservation form,

$$K_{\text{i}} + U_{\text{Gi}} + U_{\text{Sp i}} = K_{\text{f}} + U_{\text{Gf}} + U_{\text{Sp f}} + \Delta E_{\text{th}} \tag{10.22}$$

This is looking a bit more complex as we have more and more energies to keep track of, but the message of Equations 10.21 and 10.22 is both simple and profound: **For a system that has no other interactions with its environment, the total energy of the system does not change.** It can be transformed in many ways by the interactions, but the total does not change.

EXAMPLE 10.6 A spring-launched block

Your lab assignment for the week is to devise an innovative method to determine the spring constant of a spring. You see several small blocks of different mass lying around, so you decide to measure how high the compressed spring will launch each of the blocks. You and your lab partners realize that you need to compress the spring the same amount each time, so that only the mass is varying, and you choose to use a compression of 4.0 cm. You decide to measure height from the point on the compressed spring at which the block is released. Four launches generate the data in the table:

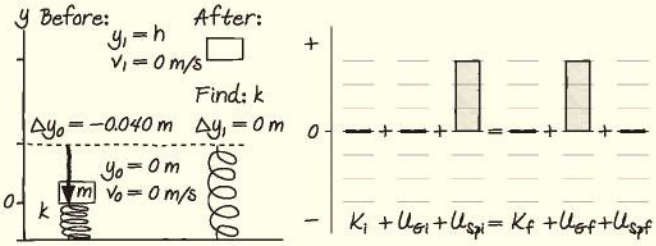
Mass (g)	Height (m)
50	2.07
100	1.11
150	0.65
200	0.51

What value will you report for the spring constant?

MODEL Let the system consist of the earth, the block, the spring, and the floor, so there will be two potential energies. There's no friction and we'll assume no drag; hence the mechanical energy of this system is conserved. Model the spring as ideal.

VISUALIZE FIGURE 10.13 shows a pictorial representation, including an energy bar chart. We've chosen to place the origin of the

FIGURE 10.13 Pictorial representation of the experiment.



Continued

coordinate system at the point of launch. The projectile reaches height $y_1 = h$, at which point $v_1 = 0$ m/s.

SOLVE You might think we would need to find the block's speed as it leaves the spring. That would be true if we were solving this problem with Newton's laws of motion. But with an energy analysis, we can compare the system's pre-launch energy to its energy when the block reaches its highest point, completely bypassing the launch speed. The block certainly has kinetic energy *during* the motion, but the net energy transfer, shown on the energy bar chart, is from elastic potential energy to gravitational potential energy.

With both elastic and gravitational potential energy included, the energy principle is

$$\begin{aligned} K_\mathrm{i} + U_\mathrm{Gi} + U_\mathrm{Sp\,i} &= 0 + 0 + \tfrac{1}{2}k(\Delta y_0)^2 \\ &= K_\mathrm{f} + U_\mathrm{Gf} + U_\mathrm{Sp\,f} = 0 + mgy_1 + 0 \end{aligned}$$

The block travels to position y_1, but the end of the spring does not! Be careful in spring problems not to mistake the position of an object for the position of the end of the spring; sometimes they are the same, but not always. Here the final elastic potential energy is that of an empty, unstretched spring: zero. Solving for the height, we find

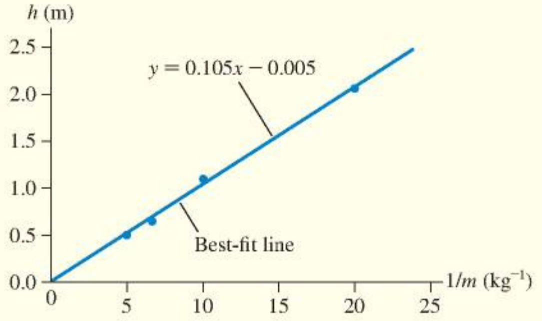
$$y_1 = h = \frac{k(\Delta y_0)^2}{2mg} = \frac{k(\Delta y_0)^2}{2g} \times \frac{1}{m}$$

The first expression is correct as an algebraic expression, but here we want to use the result to analyze an experiment in which we measure h as m is varied. By isolating the mass term, we see that plotting h versus $1/m$ (that is, using $1/m$ as the x-variable) should yield a straight line with slope $k(\Delta y_0)^2/2g$.

FIGURE 10.14 is a graph of h versus $1/m$, with masses first converted to kg. The graph is linear and the best-fit line has a y-intercept very near zero, confirming our analysis of the situation. The experimentally determined slope is 0.105 m kg, with the units determined by rise over run. Thus the experimental value of the spring constant is

$$k = \frac{2g}{(\Delta y_0)^2} \times \text{slope} = 1290 \text{ N/m}$$

FIGURE 10.14 Graph of the block height versus the inverse of its mass.



ASSESS 1290 N/m is a reasonably stiff spring, but that's to be expected if you're launching blocks a meter or more into the air.

STOP TO THINK 10.5 A spring-loaded pop gun shoots a plastic ball with a speed of 4 m/s. If the spring is compressed twice as far, the ball's speed will be

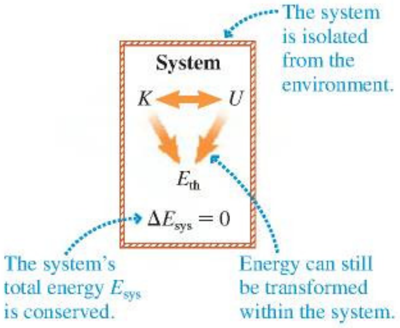
a. 2 m/s
b. 4 m/s
c. 8 m/s
d. 16 m/s

10.4 Conservation of Energy

One of the most powerful statements in physics is the **law of conservation of energy:**

> **Law of conservation of energy** The total energy $E_\mathrm{sys} = E_\mathrm{mech} + E_\mathrm{th}$ of an isolated system is a constant. The kinetic, potential, and thermal energy within the system can be transformed into each other, but their sum cannot change. Further, the mechanical energy $E_\mathrm{mech} = K + U$ is conserved if the system is both isolated and nondissipative.

FIGURE 10.15 The basic energy model for an isolated system.



The system is isolated from the environment.

The system's total energy E_sys is conserved.

Energy can still be transformed within the system.

The key is that energy is conserved for an **isolated system,** a system that does not exchange energy with its environment either because it has no interactions with the environment or because those interactions do no work. **FIGURE 10.15** shows our basic energy model for an isolated system.

It Depends on the System

As significant as the law of conservation of energy is, it's critical to notice that the law does *not* say "Energy is always conserved." The law of conservation of energy refers to the energy *of a system*—hence our emphasis on systems in Chapters 9 and 10. Energy is conserved for some choices of system, but not others. For example, energy is conserved for a projectile moving near the earth if you define the system to be projectile + earth, but not if you define the system to be only the projectile.

In addition, the law of conservation of energy comes with an important qualification: Is the system isolated? Energy is certainly not conserved if an external force does work on the system. Thus the answer to the question "Is energy conserved?" is "It depends on the system."

A Strategy for Energy Problems

This is a good place to summarize the problem-solving strategy we've been developing for using the law of conservation of energy.

PROBLEM-SOLVING STRATEGY 10.1

Energy-conservation problems

MODEL Define the system so that there are no external forces or so that any external forces do no work on the system. If there's friction, bring both surfaces into the system. Model objects as particles and springs as ideal.

VISUALIZE Draw a before-and-after pictorial representation and an energy bar chart. A free-body diagram may be needed to visualize forces.

SOLVE If the system is both isolated and nondissipative, then the mechanical energy is conserved:

$$K_i + U_i = K_f + U_f$$

where K is the total kinetic energy of all moving objects and U is the total potential energy of all interactions within the system. If there's friction, then

$$K_i + U_i = K_f + U_f + \Delta E_{th}$$

where the thermal energy increase due to friction is $\Delta E_{th} = f_k \Delta s$.

ASSESS Check that your result has correct units and significant figures, is reasonable, and answers the question.

Exercise 14

EXAMPLE 10.7 The speed of a pendulum

A pendulum is created by attaching one end of a 78-cm-long string to the ceiling and tying a 150 g steel ball to the other end. The ball is pulled back until the string is 60° from vertical, then released. What is the speed of the ball at its lowest point?

MODEL Let the system consist of the earth and the ball. The tension force, like a normal force, is always perpendicular to the motion and does no work, so this is an isolated system with no friction. Its mechanical energy is conserved.

VISUALIZE FIGURE 10.16 shows a before-and-after pictorial representation, where we've placed the zero of potential energy at the lowest point of the ball's swing. Trigonometry is needed to determine the ball's initial height.

SOLVE Conservation of mechanical energy is

$$K_i + U_{Gi} = 0 + mgy_0 = K_f + U_{Gf} = \tfrac{1}{2}mv_1^2 + 0$$

Thus the ball's speed at the bottom is

$$v_1 = \sqrt{2gy_0} = \sqrt{2(9.80\text{ m/s}^2)(0.39\text{ m})} = 2.8\text{ m/s}$$

The speed is exactly the same as if the ball had simply fallen 0.39 m.

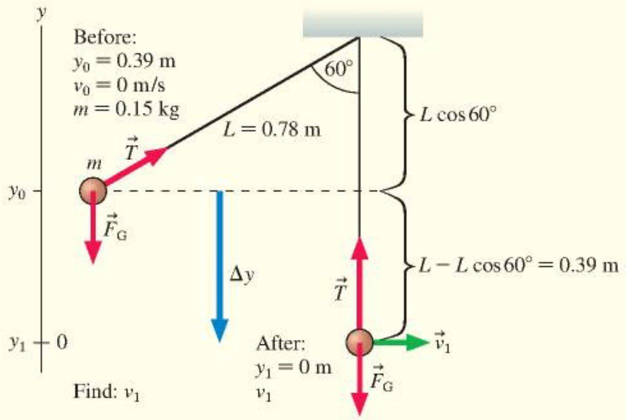


FIGURE 10.16 Pictorial representation of a pendulum.

ASSESS To solve this problem directly from Newton's laws of motion requires advanced mathematics because of the complex way the net force changes with angle. But we can solve it in one line with an energy analysis!

Where Is Potential Energy?

Kinetic energy is the energy of a moving object. The basic energy model says that kinetic energy can be transformed into potential energy without loss, but where *is* the potential energy? If energy is real, not just an accounting fiction, what is it that has potential energy?

Potential energy is stored in *fields*. We've not yet introduced fields in this textbook, although we'll have a lot to say about electric and magnetic fields in later chapters. Even so, you've no doubt heard of magnetic fields and gravitational fields. Our modern understanding of the fundamental forces of nature, the long-range forces such as gravitational and electric forces, is that they are mediated by fields. How do two masses exert forces on each other through empty space? Or two electric charges? Through their fields!

When two masses move apart, the gravitational field changes to a new configuration that can store more energy. Thus the phrase "kinetic energy is transformed into gravitational potential energy" really means that the energy of a moving object is transformed into the energy of the gravitational field. At a later time, the field's energy can be transformed back into kinetic energy. The same holds true for the energy of charges and electric fields, a topic we'll take up in Part VII.

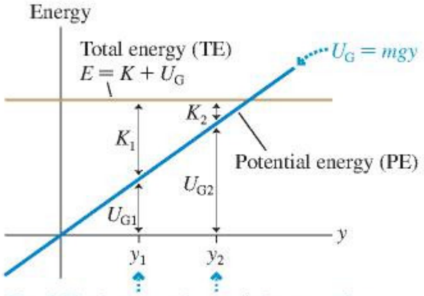
What about elastic potential energy? Remember that all solids, including springs, are held together by molecular bonds. Although quantum physics is needed for a complete understanding of bonds, they are essentially electric forces between neighboring atoms. When a solid is placed under tension, a vast number of molecular bonds stretch just a little and more energy is stored in their electric fields. What we call elastic potential energy at the macroscopic level is really energy stored in the electric fields of molecular bonds.

The theory of field energy is an advanced topic in physics. Nonetheless, this brief discussion helps complete our picture of what energy is and how it's associated with physical objects.

10.5 Energy Diagrams

Potential energy is an energy of position. The gravitational potential energy depends on the height of an object, and the elastic potential energy depends on a spring's displacement. Other potential energies you will meet in the future will depend in some way on position. Functions of position are easy to represent as graphs. A graph showing a system's potential energy and total energy as a function of position is called an **energy diagram.** Energy diagrams allow you to visualize motion based on energy considerations.

FIGURE 10.17 The energy diagram of a particle in free fall.



K and U_G change as the particle moves from y_1 to y_2, but their sum is always E.

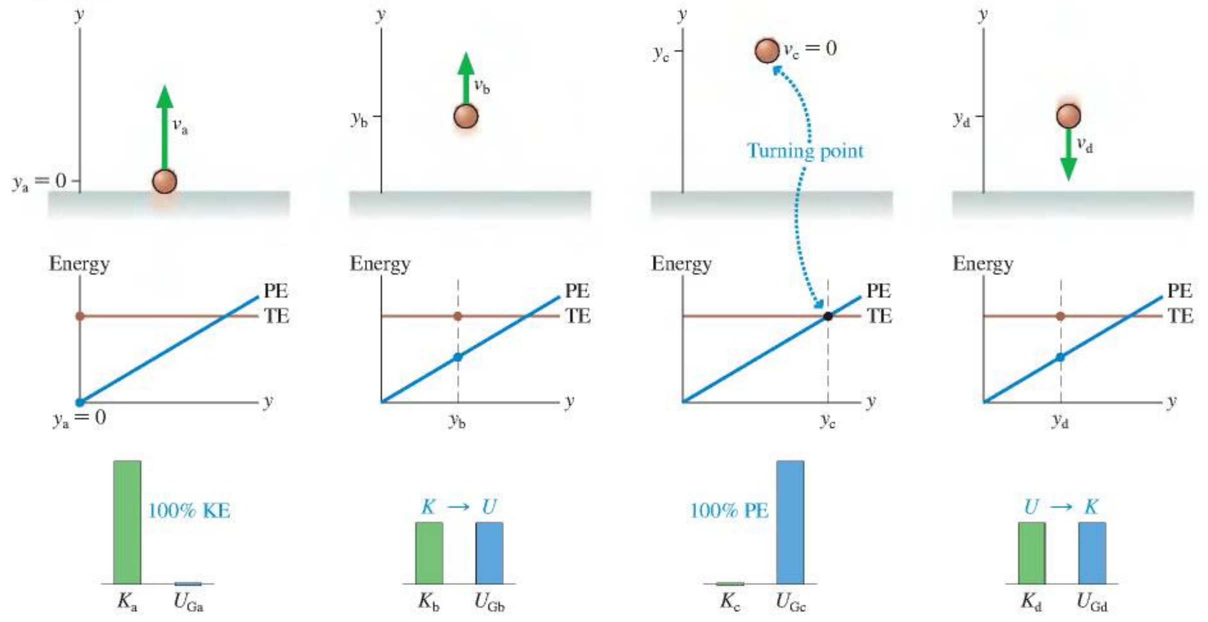
FIGURE 10.17 is the energy diagram of a particle in free fall. The gravitational potential energy $U_G = mgy$ is graphed as a line through the origin with slope mg. The *potential-energy curve* is labeled PE. The line labeled TE is the *total energy line*, $E = K + U_G$. It is horizontal because mechanical energy is conserved, meaning that the object's mechanical energy E has the same value at every position.

Suppose the particle is at position y_1. By definition, the distance from the axis to the potential-energy curve is the system's potential energy U_{G1} at that position. Because $K_1 = E - U_{G1}$, the distance between the potential-energy curve and the total energy line is the particle's kinetic energy.

The four-frame "movie" of **FIGURE 10.18** illustrates how an energy diagram is used to visualize motion. The first frame shows a particle projected upward from $y_a = 0$ with kinetic energy K_a. Initially the energy is entirely kinetic, with $U_{Ga} = 0$. A pictorial representation and an energy bar chart help to illustrate what the energy diagram is showing.

In the second frame, the particle has gained height but lost speed. The potential energy U_{Gb} is larger, and the distance K_b between the potential-energy curve and the total energy line is less. The particle continues rising and slowing until, in the third frame, it reaches the y-value where the total energy line crosses the potential-energy

FIGURE 10.18 A four-frame movie of a particle in free fall.



curve. This point, where $K = 0$ and the energy is entirely potential, is a *turning point* where the particle reverses direction. Finally, we see the particle speeding up as it falls.

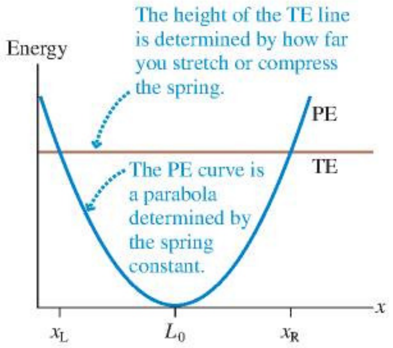
A particle with this amount of total energy would need negative kinetic energy to be to the right of the point, at y_c, where the total energy line crosses the potential-energy curve. Negative K is not physically possible, so **the particle cannot be at positions with $U > E$.** Now, it's certainly true that you could make the particle reach a larger value of y simply by throwing it harder. But that would increase E and move the total energy line higher.

NOTE It's important to realize that the TE line is under your control. If you project an object with a different speed, or drop it from a different height, you're giving it a different total energy. You can give the object different *initial conditions* and use the energy diagram to explore how it will move with that amount of total energy.

FIGURE 10.19 shows the energy diagram of a mass on a horizontal spring, where x has been measured from the wall where the spring is attached. The equilibrium length of the spring is L_0 and the displacement of the end of the spring is $\Delta x = x - L_0$, so the spring's potential energy is $U_{Sp} = \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}k(x - L_0)^2$. The potential-energy curve, a graph of U_{Sp} versus x, is a parabola centered at the equilibrium position. You can't change the PE curve—it's determined by the spring constant—but you can set the TE to any height you wish simply by stretching the spring to the proper length. The figure shows one possible TE line.

If you pull the mass out to position x_R and release it, the initial mechanical energy is entirely potential. As the restoring force of the spring pulls the mass to the left, the kinetic energy increases as the potential energy decreases. The mass has maximum speed at $x = L_0$, where $U_{Sp} = 0$, and then it slows down as the spring starts to compress. You should be able to visualize that x_L, where the PE curve crosses the TE line, is a turning point. It's the point of maximum compression where the mass instantaneously has $K = 0$. The mass will reverse direction, speed up until $x = L_0$, then slow down until reaching x_R, where it started. This is another turning point, so it will reverse direction again and start the process over. In other words, the mass will *oscillate* back and forth between the left and right turning points at x_L and x_R where the TE line crosses the PE curve. We'll study oscillations in Chapter 15, but we can already see from the energy diagram that a mass on a spring undergoes oscillatory motion.

FIGURE 10.19 The energy diagram of a mass on a horizontal spring.


The height of the TE line is determined by how far you stretch or compress the spring.

The PE curve is a parabola determined by the spring constant.

FIGURE 10.20 A more general energy diagram.

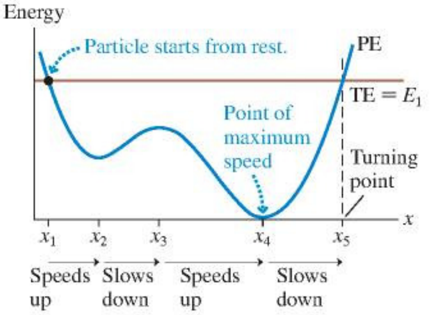
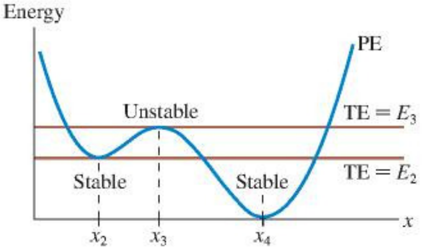


FIGURE 10.20 applies these ideas to a more general energy diagram. We don't know how this potential energy was created, but we can visualize the motion of a particle in a system that has this potential energy. Suppose the particle is released from rest at position x_1. How will it then move?

The initial conditions are $K = 0$ at x_1, hence the TE line must cross the PE curve at this point. The particle cannot move to the left, because that would require $K < 0$, so it begins to move toward the right. We see from the energy diagram that U decreases from x_1 to x_2, so the particle is speeding up as potential energy is transformed into kinetic energy. The particle then slows down from x_2 to x_3 as it goes up the "potential-energy hill," increasing U at the expense of K. The particle doesn't stop at x_3 because it still has kinetic energy. It speeds up from x_3 to x_4 (K increasing as U decreases), reaching its maximum speed at x_4, then slows down between x_4 and x_5. Position x_5 is a turning point, a point where the TE line crosses the PE curve. The particle is instantaneously at rest, then reverses direction. The particle will oscillate back and forth between x_1 and x_5, following the pattern of slowing down and speeding up that we've outlined.

Equilibrium Positions

FIGURE 10.21 Points of stable and unstable equilibrium.



Positions x_2, x_3, and x_4 in Figure 10.20, where the potential energy has a local minimum or maximum, are special positions. Consider a particle with the total energy E_2 shown in FIGURE 10.21. The particle can be at rest at x_2, with $K = 0$, but it cannot move away from x_2. In other words, a particle with energy E_2 is in *equilibrium* at x_2. If you disturb the particle, giving it a small kinetic energy and a total energy just *slightly* larger than E_2, the particle will undergo a very small oscillation centered on x_2, like a marble in the bottom of a bowl. An equilibrium for which small disturbances cause small oscillations is called a point of **stable equilibrium.** You should recognize that *any* minimum in the PE curve is a point of stable equilibrium. Position x_4 is also a point of stable equilibrium, in this case for a particle with $E = 0$.

Figure 10.21 also shows a particle with energy E_3 that is tangent to the curve at x_3. If a particle is placed *exactly* at x_3, it will stay there at rest $(K = 0)$. But if you disturb the particle at x_3, giving it an energy only slightly more than E_3, it will speed up as it moves away from x_3. This is like trying to balance a marble on top of a hill. The slightest displacement will cause the marble to roll down the hill. A point of equilibrium for which a small disturbance causes the particle to move away is called a point of **unstable equilibrium.** Any maximum in the PE curve, such as x_3, is a point of unstable equilibrium.

We can summarize these lessons as follows:

TACTICS BOX 10.1

Interpreting an energy diagram

❶ The distance from the axis to the PE curve is the particle's potential energy. The distance from the PE curve to the TE line is its kinetic energy. These are transformed as the position changes, causing the particle to speed up or slow down, but the sum $K + U$ doesn't change.

❷ A point where the TE line crosses the PE curve is a turning point. The particle reverses direction.

❸ The particle cannot be at a point where the PE curve is above the TE line.

❹ The PE curve is determined by the properties of the system—mass, spring constant, and the like. You cannot change the PE curve. However, you can raise or lower the TE line simply by changing the initial conditions to give the particle more or less total energy.

❺ A minimum in the PE curve is a point of stable equilibrium. A maximum in the PE curve is a point of unstable equilibrium.

Exercises 15–17

EXAMPLE 10.8 Balancing a mass on a spring

A spring of length L_0 and spring constant k is standing on one end. A block of mass m is placed on the spring, compressing it. What is the length of the compressed spring?

MODEL Assume an ideal spring obeying Hooke's law. The block + earth + spring system has both gravitational potential energy U_G *and* elastic potential energy U_{Sp}. The block sitting on top of the spring is at a point of stable equilibrium (small disturbances cause the block to oscillate slightly around the equilibrium position), so we can solve this problem by looking at the energy diagram.

VISUALIZE **FIGURE 10.22a** is a pictorial representation. We've used a coordinate system with the origin at ground level, so the displacement of the spring is $y - L_0$.

SOLVE **FIGURE 10.22b** shows the two potential energies separately and also shows the total potential energy:

$$\begin{aligned} U_{tot} &= U_G + U_{Sp} \\ &= mgy + \tfrac{1}{2}k(y - L_0)^2 \end{aligned}$$

The equilibrium position (the minimum of U_{tot}) has shifted from L_0 to a smaller value of y, closer to the ground. We can find the equilibrium by locating the position of the minimum in the PE curve. You know from calculus that the minimum of a function is at the point where the derivative (or slope) is zero. The derivative of U_{tot} is

$$\frac{dU_{tot}}{dy} = mg + k(y - L_0)$$

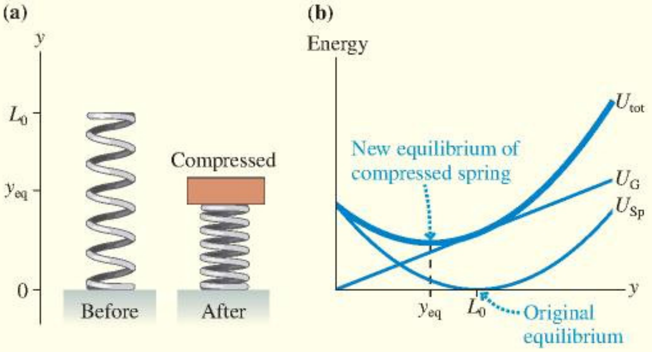
The derivative is zero at the point y_{eq}, so we can easily find

$$mg + k(y_{eq} - L_0) = 0$$

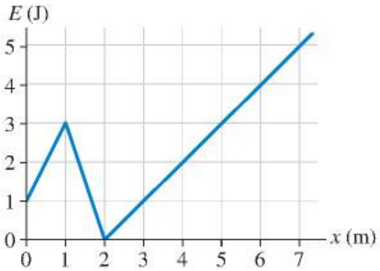
$$y_{eq} = L_0 - \frac{mg}{k}$$

The block compresses the spring by the length mg/k from its original length L_0, giving it a new equilibrium length $L_0 - mg/k$.

FIGURE 10.22 The block + earth + spring system has both gravitational and elastic potential energy.



STOP TO THINK 10.6 A particle with the potential energy shown in the graph is moving to the right. It has 1 J of kinetic energy at $x = 1$ m. Where is the particle's turning point?



10.6 Force and Potential Energy

As you've seen, we can find the energy of an interaction—potential energy—by calculating the work the interaction force does inside the system. Can we reverse this procedure? That is, if we know a system's potential energy, can we find the interaction force?

We defined the change in potential energy to be $\Delta U = -W_{int}$. Suppose that an object undergoes a *very small* displacement Δs, so small that the interaction force $\vec{F}$ is essentially constant. The work done by a constant force is $W = F_s \Delta s$, where F_s is the force component parallel to the displacement. During this small displacement, the system's potential energy changes by

$$\Delta U = -W_{int} = -F_s \Delta s \qquad (10.23)$$

FIGURE 10.23 Relating force to the PE curve.

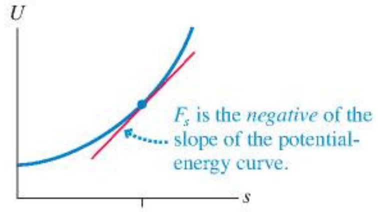


FIGURE 10.24 Elastic potential energy and force graphs.

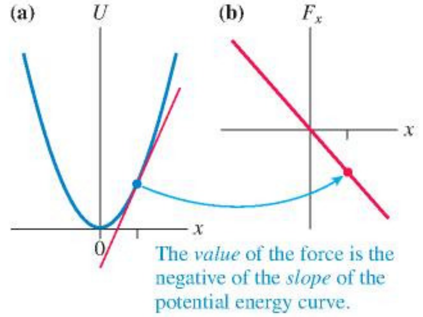
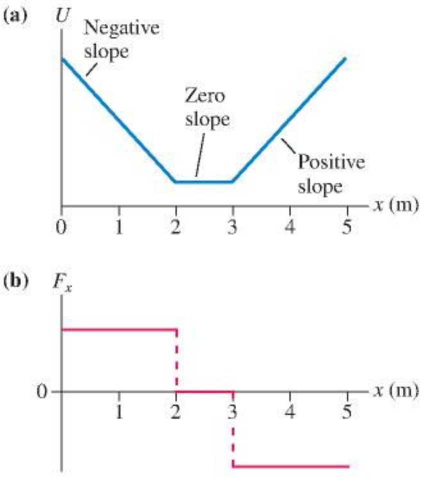


FIGURE 10.25 A potential-energy curve and the associated force curve.



which we can rewrite as

$$F_s = -\frac{\Delta U}{\Delta s} \tag{10.24}$$

In the limit $\Delta s \rightarrow 0$, the force on the object is

$$F_s = \lim_{\Delta s \rightarrow 0}\left(-\frac{\Delta U}{\Delta s}\right) = -\frac{dU}{ds} \tag{10.25}$$

That is, the interaction force on an object is the *negative* of the derivative of the potential energy with respect to position.

Graphically, as **FIGURE 10.23** shows, force is the negative of the slope, at position s, of the potential-energy curve in an energy diagram:

$$F_s = -\frac{dU}{ds} = \text{the negative of the slope of the PE curve at } s \tag{10.26}$$

In practice, of course, we'll usually use either $F_x = -dU/dx$ or $F_y = -dU/dy$. Thus

- A positive slope corresponds to a negative force: to the left or downward.
- A negative slope corresponds to a positive force: to the right or upward.
- The steeper the slope, the larger the force.

As an example, consider the elastic potential energy $U_{\text{Sp}} = \frac{1}{2}kx^2$ for a horizontal spring with $x_{\text{eq}} = 0$ so that $\Delta x = x$. **FIGURE 10.24a** shows that the potential-energy curve is a parabola, with changing slope. If an object attached to the spring is at position x, the force on the object is

$$F_x = -\frac{dU_{\text{Sp}}}{dx} = -\frac{d}{dx}\left(\tfrac{1}{2}kx^2\right) = -kx$$

This is just Hooke's law for an ideal spring, with the minus sign indicating that Hooke's law is a restoring force. **FIGURE 10.24b** is a graph of force versus x. At each position x, the *value* of the force is equal to the negative of the *slope* of the PE curve.

We already knew Hooke's law, of course, so the point of this particular exercise was to illustrate the meaning of Equation 10.26. But if we had *not* known the force, we see that it's possible to find the force from the PE curve. For example, you'll learn in Part VIII that **FIGURE 10.25a** is a possible potential-energy function for a charged particle, one that we could create with suitably shaped electrodes. What force does the particle experience in this region of space? We find out by measuring the slope of the PE curve. The result is shown in **FIGURE 10.25b**. On the left side of this region of space ($x < 2$ m), a negative slope, and thus a positive value of F_x, means that the force pushes the particle to the right. A negative force on the right side ($x > 3$ m) tells us that F_x pushes the particle to the left. And there's no force at all in the center. This is a *restoring force* because a particle trying to leave this region is pushed back toward the center, but it's not a linear restoring force like that of a spring.

EXAMPLE 10.9 Finding equilibrium positions

A system's potential energy is given by $U(x) = (2x^3 - 3x^2)$ J, where x is a particle's position in m. Where are the equilibrium positions for this system, and are they stable or unstable equilibria?

SOLVE You learned in Chapter 6 that a particle in equilibrium has $\vec{F}_{\text{net}} = \vec{0}$. Then, in the previous section, you learned that the maxima and minima of the PE curve are points of equilibrium.

These may seem to be two different criteria for equilibrium, but actually they are identical. The interaction force on the particle is $F_x = -dU/dx$. The force is zero—equilibrium—at positions where the derivative is zero. But you've learned in calculus that positions where the derivative of a function is zero are the maxima and minima of the function. At either a maximum or minimum of the PE curve, the slope is zero and hence the force is zero.

For this potential-energy function,

$$F_x = -\frac{dU}{dx} = (-6x^2 + 6x)\ \text{N}$$

when x is in m. The force is zero—minima or maxima of U—when $6x_{\text{eq}}{}^2 = 6x_{\text{eq}}$. This has two solutions:

$$x_{\text{eq}} = 0\ \text{m} \quad \text{and} \quad x_{\text{eq}} = 1\ \text{m}$$

These are positions of equilibrium, where a particle at rest will remain at rest. But how do we know if these are positions of stable equilibrium or unstable equilibrium?

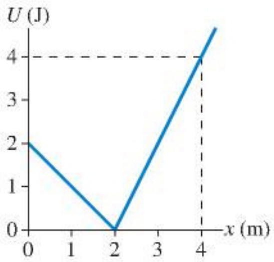
A *minimum* in the PE curve is a stable equilibrium. Recall, from calculus, that a minimum of a function has a first derivative equal to zero *and* a second derivative that's positive. Similarly, a maximum of a function has a first derivative equal to zero and a second derivative that's negative. The second derivative of U is

$$\frac{d^2U}{dx^2} = \frac{d}{dx}(6x^2 - 6x) = (12x - 6)\ \text{N/m}$$

The second derivative evaluated at $x = 0$ m is -6 N/m < 0, so $x = 0$ m is a maximum of the PE curve. At $x = 1$ m, the second derivative is $+6$ N/m > 0, hence a minimum in the PE curve. Thus this system has an unstable equilibrium if the particle is at $x = 0$ m and a stable equilibrium if it is at $x = 1$ m. There's a force on the particle at all other positions.

STOP TO THINK 10.7 A particle moves along the x-axis with the potential energy shown. The x-component of the force on the particle when it is at $x = 4$ m is

a. 4 N
b. 2 N
c. 1 N
d. -4 N
e. -2 N
f. -1 N



10.7 Conservative and Nonconservative Forces

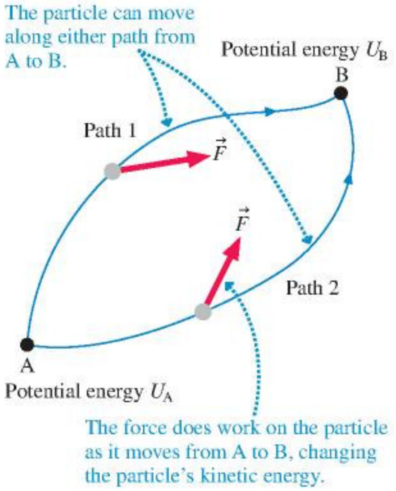
A system in which particles interact gravitationally or elastically or, as we'll discover later, electrically has a potential energy. But do all forces have potential energies? Is there a "tension potential energy" or a "friction potential energy"? If not, what's special about the gravitational and elastic forces? What conditions must an interaction satisfy to have an associated potential energy?

FIGURE 10.26 shows a particle that can move from point A to point B along two possible paths while a force $\vec{F}$ acts on it. In general, the force experienced along path 1 is not the same as the force experienced along path 2. The force changes the particle's speed, so the particle's kinetic energy when it arrives at B differs from the kinetic energy it had when it left A.

Let's assume that there is a potential energy U associated with force $\vec{F}$ just as the gravitational potential energy $U_G = mgy$ is associated with the gravitational force $\vec{F}_G = -mg\hat{j}$. What restrictions does this assumption place on $\vec{F}$? There are three steps in the logic:

1. Potential energy is an energy of position. U depends only on where the particle is, not on how it got there. The system has one value of potential energy when the particle is at A, a different value when the particle is at B. Thus the change in potential energy, $\Delta U = U_B - U_A$, is the same whether the particle moves along path 1 or path 2.

FIGURE 10.26 A particle can move from A to B along either of two paths.



2. Potential energy is transformed into kinetic energy, with $\Delta K = -\Delta U$. If ΔU is independent of the path followed, then ΔK is also independent of the path. The particle has the same kinetic energy at B no matter which path it follows.
3. According to the energy principle, the change in a particle's kinetic energy is equal to the work done on the particle by force $\vec{F}$. That is, $\Delta K = W$. Because ΔK is independent of the path followed, it *must* be the case that **the work done by force $\vec{F}$ as the particle moves from A to B is independent of the path followed.**

A force for which the work done on a particle as it moves from an initial to a final position is independent of the path followed is called a **conservative force.** The importance of conservative forces is that **a potential energy can be associated with any conservative force.** Specifically, the potential-energy difference between an initial position i and a final position f is

$$\Delta U = -W_c(\mathrm{i} \rightarrow \mathrm{f}) \tag{10.27}$$

where the notation $W_c(\mathrm{i} \rightarrow \mathrm{f})$ is the work done by a conservative force as the particle moves along *any* path from i to f. Equation 10.27 is a general definition of the potential energy associated with a conservative force.

A force for which we can define a potential energy is called *conservative* because the mechanical energy $K + U$ is conserved for a system in which this is the only interaction. We've already shown that the gravitational force is a conservative force by showing that ΔU_G depends only on the vertical displacement, not on the path followed; hence mechanical energy is conserved when two masses interact gravitationally. Similarly, mechanical energy is conserved for a mass on a spring—an elastic interaction—if there are no other forces. Conservative forces do not contribute to any loss of mechanical energy.

Nonconservative Forces

A characteristic of a conservative force is that **an object returning to its starting point will return with no loss of kinetic energy** because $\Delta U = 0$ if the initial and final points are the same. If a ball is tossed into the air, energy is transformed from kinetic into potential and back such that the ball's kinetic energy is unchanged when it returns to its initial height. The same is true for a puck sliding up and back down a frictionless slope.

But not all forces are conservative forces. If the slope has friction, then the puck returns with *less* kinetic energy. Part of its kinetic energy is transformed into gravitational potential energy as it slides up, but part is transformed into some other form of energy—thermal energy—that lacks the "potential" to be transformed back into kinetic energy. A force for which we cannot define a potential energy is called a **nonconservative force.** Friction and drag, which transform mechanical energy into thermal energy, are nonconservative forces, so there is no "friction potential energy."

Similarly, forces like tension and thrust are nonconservative. If you pull an object with a rope, the work done by tension is proportional to the distance traveled. More work is done along a longer path between two points than along a shorter path, so tension fails the "Work is independent of the path followed" test and does not have a potential energy.

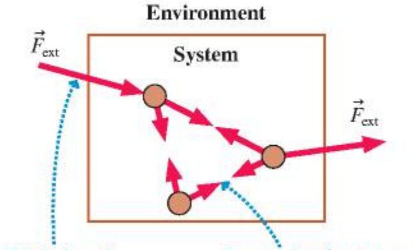
All in all, most forces are *not* conservative forces. Gravitational forces, linear restoring forces, and, later, electric forces turn out to be fairly special because they are among the few forces for which we *can* define a potential energy. Fortunately, these are some of the most important forces in nature, so the energy principle is powerful and useful despite there being only a small number of conservative forces.

10.8 The Energy Principle Revisited

We opened Chapter 9 by introducing the energy principle—basically a statement of energy accounting—but noted that we would need to develop many new ideas to make sense of energy. We've now explored kinetic energy, potential energy, work, conservative and nonconservative forces, and much more. It's time to return to the basic energy model and start pulling together the many ideas introduced in Chapters 9 and 10.

FIGURE 10.27 shows a system of three objects that interact with each other and are acted on by external forces from the environment. These forces cause the system's kinetic energy K to change. By how much? Kinetic energy is energy of motion, and the kinetic energy would be the same if we had defined the system—as we did in Chapter 9— to consist of only the objects, not the interactions. Thus $\Delta K = W_{\text{tot}} = W_c + W_{\text{nc}}$, where in the second step we've divided the total work done by all forces into the work W_c done by conservative forces and the work W_{nc} done by nonconservative forces.

FIGURE 10.27 A system with both internal interactions and external forces.



Work done by forces from the environment is external work W_{ext}.

Interaction forces can be either conservative or dissipative. These do work W_c and W_{diss}.

Now let's make a further distinction by dividing the nonconservative forces into *dissipative* forces and *external* forces. Dissipative forces, like friction and drag, transform mechanical energy into thermal energy.

To illustrate what we mean by an external force, suppose you pick up a box at rest on the floor and place it at rest on a table. The box + earth system gains gravitational potential energy, but $\Delta K = 0$ and $\Delta E_{\text{th}} = 0$. So where did the energy come from? Or consider pulling the box across the table with a string. The box gains kinetic energy and possibly thermal energy, but not by transforming potential energy. The force of your hand and the tension of the string are forces that "reach in" from the environment to change the system. Thus they are *external forces*. They are nonconservative forces, with no potential energy, but they change the system's mechanical energy rather than its thermal energy.

With this distinction, the system's change in kinetic energy is

$$\Delta K = W_{\text{tot}} = W_c + W_{\text{nc}} = W_c + W_{\text{diss}} + W_{\text{ext}} \tag{10.28}$$

When we bring the conservative interactions inside the system, the work done by conservative forces becomes potential energy: $W_c = -\Delta U$. And, as we learned in Chapter 9, the work done by dissipative forces becomes thermal energy: $W_{\text{diss}} = -\Delta E_{\text{th}}$. With these substitutions, Equation 10.28 becomes

$$\Delta K = -\Delta U - \Delta E_{\text{th}} + W_{\text{ext}} \tag{10.29}$$

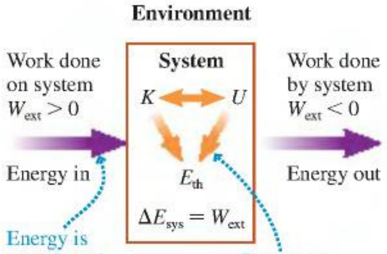
Separating energy terms from the work, we can write Equation 10.29 as

$$\Delta K + \Delta U + \Delta E_{\text{th}} = \Delta E_{\text{mech}} + \Delta E_{\text{th}} = \Delta E_{\text{sys}} = W_{\text{ext}} \tag{10.30}$$

where $E_{\text{sys}} = K + U + E_{\text{th}} = E_{\text{mech}} + E_{\text{th}}$ is the energy of the system. Equation 10.30, the energy principle but with all the terms now defined, is our most general statement about how the energy of a mechanical system changes.

In Section 10.4 we defined an *isolated system* as a system that does not exchange energy with its environment. That is, an isolated system is one on which no work is done by external forces: $W_{\text{ext}} = 0$. Thus an immediate conclusion from Equation 10.30 is that **the total energy E_{sys} of an isolated system is conserved.** If, in addition, the system is nondissipative (i.e., no friction forces), then $\Delta E_{\text{th}} = 0$. In that case, the mechanical energy E_{mech} is conserved. You'll recognize this as the *law of conservation of energy* from Section 10.4. The law of conservation of energy is one of the most powerful statements in physics.

FIGURE 10.28 The basic energy model.



Energy is *transferred* to (and from) the system.

Energy is *transformed* within the system.

FIGURE 10.28 reproduces the basic energy model of Chapter 9. Now you can see that this is a pictorial representation of Equation 10.30. E_{sys}, the total energy of the system, changes only if external forces transfer energy into or out of the system by doing work on the system. The kinetic, potential, and thermal energies within the system can be transformed into each other by forces within the system. And E_{sys} is conserved in the absence of interactions with the environment.

Energy Bar Charts Expanded

Energy bar charts can now be expanded to include the thermal energy and the work done by external forces. The energy principle, Equation 10.30, can be rewritten as

$$K_i + U_i + W_{\text{ext}} = K_f + U_f + \Delta E_{\text{th}} \tag{10.31}$$

The initial mechanical energy $(K_i + U_i)$ plus any energy added to or removed from the system (W_{ext}) becomes, without loss, the final mechanical energy $(K_f + U_f)$ plus any increase in the system's thermal energy (ΔE_{th}). Remember that we have no way to determine $E_{\text{th i}}$ or $E_{\text{th f}}$, only the *change* in thermal energy. ΔE_{th} is always positive when the system contains dissipative forces.

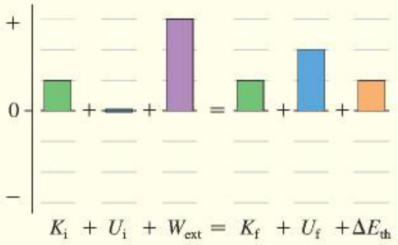
EXAMPLE 10.10 Hauling up supplies

A mountain climber uses a rope to drag a bag of supplies up a slope at constant speed. Show the energy transfers and transformations on an energy bar chart.

MODEL Let the system consist of the earth, the bag of supplies, and the slope.

SOLVE The tension in the rope is an external force that does work on the bag of supplies. This is an energy transfer into the system. The bag has kinetic energy, but it moves at a steady speed and so K is not *changing*. Instead, the energy transfer into the system increases both gravitational potential energy (the bag is gaining height) and thermal energy (the bag and the slope are getting warmer due to friction). The overall process is $W_{\text{ext}} \rightarrow U + E_{\text{th}}$. This is shown in **FIGURE 10.29**.

FIGURE 10.29 The energy bar chart for Example 10.10.



STOP TO THINK 10.8 A weight attached to a rope is released from rest. As the weight falls, picking up speed, the rope spins a generator that causes a lightbulb to glow. Define the system to be the weight and the earth. In this situation,

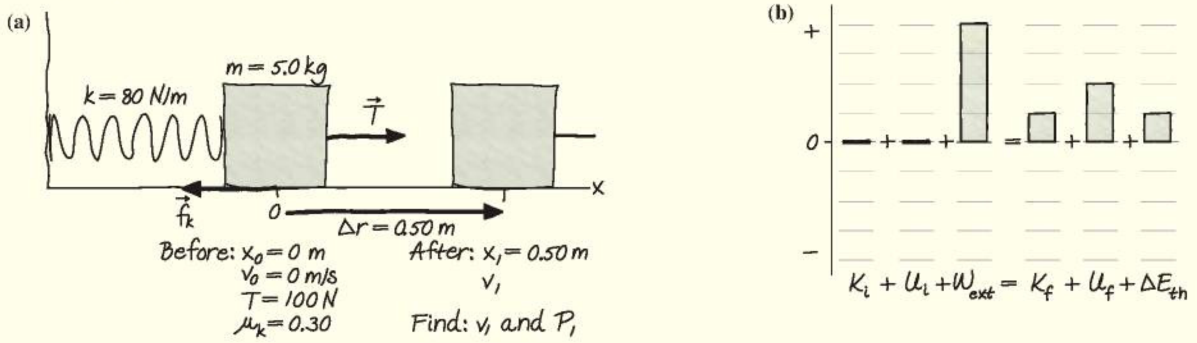
a. $U \rightarrow K + W_{\text{ext}}$. E_{mech} is not conserved but E_{sys} is.
b. $U + W_{\text{ext}} \rightarrow K$. Both E_{mech} and E_{sys} are conserved.
c. $U \rightarrow K + E_{\text{th}}$. E_{mech} is not conserved but E_{sys} is.
d. $U \rightarrow K + W_{\text{ext}}$. Neither E_{mech} nor E_{sys} is conserved.
e. $W_{\text{ext}} \rightarrow K + U$. E_{mech} is not conserved but E_{sys} is.

CHALLENGE EXAMPLE 10.11 A spring workout

An exercise machine at the gym has a 5.0 kg weight attached to one end of a horizontal spring with spring constant 80 N/m. The other end of the spring is anchored to a wall. When a woman working out on the machine pushes her arms forward, a cable stretches the spring by dragging the weight along a track with a coefficient of kinetic friction of 0.30. What is the woman's power output at the moment when the weight has moved 50 cm if the cable tension is a constant 100 N?

MODEL This is a complex situation, but one that we can analyze. First, identify the weight, the spring, the wall, and the track as the system. We need to have the track inside the system because friction increases the temperature of both the weight *and* the track. The tension in the cable is an external force. The work W_{ext} done by the cable's tension transfers energy into the system, causing K, U_{Sp}, and E_{th} all to increase.

FIGURE 10.30 Pictorial representation and energy bar chart for Challenge Example 10.11.



VISUALIZE FIGURE 10.30a is a before-and-after pictorial representation. The energy transfers and transformations are shown in the energy bar chart of FIGURE 10.30b.

SOLVE You learned in « Section 9.6 that power is the *rate* at which work is done and that the power delivered by force $\vec{F}$ to an object moving with velocity $\vec{v}$ is $P = \vec{F} \cdot \vec{v}$. Here the tension $\vec{T}$ pulls parallel to the weight's velocity, so the power being supplied when the weight has velocity $\vec{v}$ is $P = Tv$. We know the cable's tension, so we need to use energy considerations to find the weight's speed v_1 after the spring has been stretched to $\Delta x_1 = 50$ cm.

The energy principle $K_i + U_i + W_{ext} = K_f + U_f + \Delta E_{th}$ is

$$\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}k(\Delta x_0)^2 + W_{ext} = \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}k(\Delta x_1)^2 + \Delta E_{th}$$

The initial displacement is $\Delta x_0 = 0$ m and we know that $v_0 = 0$ m/s, so the energy principle simplifies to

$$\tfrac{1}{2}mv_1^2 = W_{ext} - \tfrac{1}{2}k(\Delta x_1)^2 - \Delta E_{th}$$

The external work done by the cable's tension is

$$W_{ext} = T\Delta r = (100\ \text{N})(0.50\ \text{m}) = 50.0\ \text{J}$$

From Chapter 9, the increase in thermal energy due to friction is

$$\Delta E_{th} = f_k \Delta r = \mu_k mg \Delta r$$
$$= (0.30)(5.0\ \text{kg})(9.80\ \text{m/s}^2)(0.50\ \text{m}) = 7.4\ \text{J}$$

Solving for the speed v_1, when the spring's displacement is $\Delta x_1 = 50\ \text{cm} = 0.50$ m, we have

$$v_1 = \sqrt{\frac{2(W_{ext} - \tfrac{1}{2}k(\Delta x_1)^2 - \Delta E_{th})}{m}} = 3.6\ \text{m/s}$$

The power being supplied at this instant to keep stretching the spring is

$$P = Tv_1 = (100\ \text{N})(3.6\ \text{m/s}) = 360\ \text{W}$$

ASSESS The work done by the cable's tension is energy transferred to the system. Part of the energy increases the speed of the weight, part increases the potential energy stored in the spring, and part is transformed into increased thermal energy, thus increasing the temperature. We had to bring all these energy ideas together to solve this problem.

SUMMARY

The goal of Chapter 10 has been to develop a better understanding of energy and its conservation.

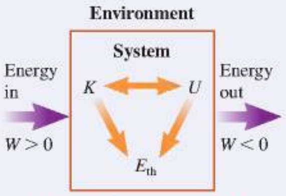
GENERAL PRINCIPLES

The Energy Principle Revisited

- Energy is *transformed* within the system.
- Energy is *transferred* to and from the system by work W.

Two variations of the energy principle are

$$\Delta E_{\text{sys}} = \Delta K + \Delta U + \Delta E_{\text{th}} = W_{\text{ext}}$$
$$K_i + U_i + W_{\text{ext}} = K_f + U_f + \Delta E_{\text{th}}$$



Basic energy model

Solving Energy Problems

MODEL Define the system.

VISUALIZE Draw a before-and-after pictorial representation and an energy bar chart.

SOLVE Use the energy principle:

$$K_i + U_i + W_{\text{ext}} = K_f + U_f + \Delta E_{\text{th}}$$

ASSESS Is the result reasonable?

Law of Conservation of Energy

- **Isolated system:** $W_{\text{ext}} = 0$. The total system energy $E_{\text{sys}} = K + U + E_{\text{th}}$ is conserved. $\Delta E_{\text{sys}} = 0$.
- **Isolated, nondissipative system:** $W_{\text{ext}} = 0$ and $W_{\text{diss}} = 0$. The **mechanical energy** $E_{\text{mech}} = K + U$ is conserved: $K_i + U_i = K_f + U_f$.

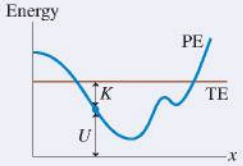
IMPORTANT CONCEPTS

Potential energy, or *interaction energy*, is energy stored inside a system via interaction forces. The energy is stored in *fields*.

- Potential energy is associated only with **conservative forces** for which the work done is independent of the path.
- Work W_{int} by the interaction forces causes $\Delta U = -W_{\text{int}}$.
- Force $F_s = -dU/ds = -$(slope of the potential energy curve).
- Potential energy is an energy of the system, not an energy of a specific object.

Energy diagrams show the potential-energy curve PE and the total mechanical energy line TE.

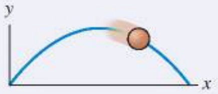
- From the axis to the curve is U. From the curve to the TE line is K.
- **Turning points** occur where the TE line crosses the PE curve.
- Minima and maxima in the PE curve are, respectively, positions of **stable** and **unstable equilibrium.**



APPLICATIONS

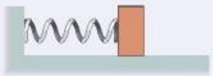
Gravitational potential energy is an energy of the earth + object system:

$$U_G = mgy$$

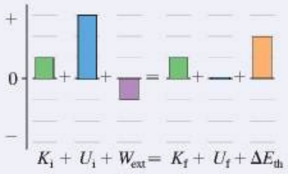


Elastic potential energy is an energy of the spring + attached objects system:

$$U_{\text{Sp}} = \tfrac{1}{2}k(\Delta s)^2$$



Energy bar charts show the energy principle in graphical form.



TERMS AND NOTATION

potential energy, U
gravitational potential energy, U_G
zero of potential energy
mechanical energy, E_{mech}
energy bar chart
elastic potential energy, U_{Sp}
law of conservation of energy
isolated system
energy diagram
stable equilibrium
unstable equilibrium
conservative force
nonconservative force

CONCEPTUAL QUESTIONS

1. Upon what basic quantity does kinetic energy depend? Upon what basic quantity does potential energy depend?
2. Can kinetic energy ever be negative? Can gravitational potential energy ever be negative? For each, give a plausible *reason* for your answer without making use of any equations.
3. A roller-coaster car rolls down a frictionless track, reaching speed v_0 at the bottom. If you want the car to go twice as fast at the bottom, by what factor must you increase the height of the track? Explain.
4. The three balls in **FIGURE Q10.4**, which have equal masses, are fired with equal speeds from the same height above the ground. Rank in order, from largest to smallest, their speeds v_a, v_b, and v_c as they hit the ground. Explain.

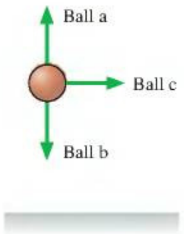


FIGURE Q10.4

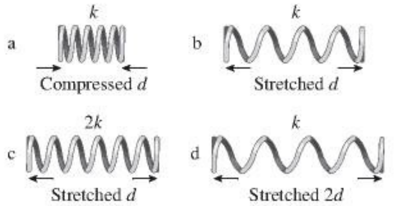


FIGURE Q10.5

5. Rank in order, from most to least, the elastic potential energy $(U_{\text{Sp}})_a$ to $(U_{\text{Sp}})_d$ stored in the springs of **FIGURE Q10.5**. Explain.
6. A spring is compressed 1.0 cm. How far must you compress a spring with twice the spring constant to store the same amount of energy?
7. A spring gun shoots out a plastic ball at speed v_0. The spring is then compressed twice the distance it was on the first shot. By what factor is the ball's speed increased? Explain.
8. A particle with the potential energy shown in **FIGURE Q10.8** is moving to the right at $x = 5$ m with total energy E.
   a. At what value or values of x is this particle's speed a maximum?
   b. Does this particle have a turning point or points in the range of x covered by the graph? If so, where?
   c. If E is changed appropriately, could the particle remain at rest at any point or points in the range of x covered by the graph? If so, where?

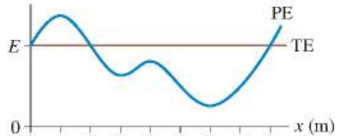


FIGURE Q10.8

9. A compressed spring launches a block up an incline. Which objects should be included within the system in order to make an energy analysis as easy as possible?
10. A process occurs in which a system's potential energy decreases while the system does work on the environment. Does the system's kinetic energy increase, decrease, or stay the same? Or is there not enough information to tell? Explain.
11. A process occurs in which a system's potential energy increases while the environment does work on the system. Does the system's kinetic energy increase, decrease, or stay the same? Or is there not enough information to tell? Explain.
12. **FIGURE Q10.12** is the energy bar chart for a firefighter sliding down a fire pole from the second floor to the ground. Let the system consist of the firefighter, the pole, and the earth. What are the bar heights of W_{ext}, K_f, and U_{Gf}?

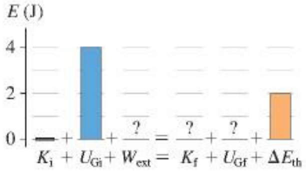


FIGURE Q10.12

13. a. If the force on a particle at some point in space is zero, must its potential energy also be zero at that point? Explain.
    b. If the potential energy of a particle at some point in space is zero, must the force on it also be zero at that point? Explain.

EXERCISES AND PROBLEMS

Problems labeled ▨ integrate material from earlier chapters.

Exercises

Section 10.1 Potential Energy

1. | Object A is stationary while objects B and C are in motion. Forces from object A do 10 J of work on object B and –5 J of work on object C. Forces from the environment do 4 J of work on object B and 8 J of work on object C. Objects B and C do not interact. What are ΔK_{tot} and ΔU_{int} if (a) objects A, B, and C are defined as separate systems and (b) one system is defined to include objects A, B, and C and their interactions?
2. | A system of two objects has $\Delta K_{\text{tot}} = 7$ J and $\Delta U_{\text{int}} = -5$ J.
   a. How much work is done by interaction forces?
   b. How much work is done by external forces?

Section 10.2 Gravitational Potential Energy

3. | The lowest point in Death Valley is 85 m below sea level. The summit of nearby Mt. Whitney has an elevation of 4420 m. What is the change in potential energy when an energetic 65 kg hiker makes it from the floor of Death Valley to the top of Mt. Whitney?
4. | a. What is the kinetic energy of a 1500 kg car traveling at a speed of 30 m/s ($\approx$65 mph)?
   b. From what height would the car have to be dropped to have this same amount of kinetic energy just before impact?
5. | a. With what minimum speed must you toss a 100 g ball straight up to just touch the 10-m-high roof of the gymnasium if you release the ball 1.5 m above the ground? Solve this problem using energy.
   b. With what speed does the ball hit the ground?

6. | What height does a frictionless playground slide need so that a 35 kg child reaches the bottom at a speed of 4.5 m/s?

7. | A 55 kg skateboarder wants to just make it to the upper edge of a "quarter pipe," a track that is one-quarter of a circle with a radius of 3.0 m. What speed does he need at the bottom?

8. | What minimum speed does a 100 g puck need to make it to the top of a 3.0-m-long, 20° frictionless ramp?

9. || A pendulum is made by tying a 500 g ball to a 75-cm-long string. The pendulum is pulled 30° to one side, then released. What is the ball's speed at the lowest point of its trajectory?

10. || A 20 kg child is on a swing that hangs from 3.0-m-long chains. What is her maximum speed if she swings out to a 45° angle?

11. || A 1500 kg car traveling at 10 m/s suddenly runs out of gas while approaching the valley shown in **FIGURE EX10.11**. The alert driver immediately puts the car in neutral so that it will roll. What will be the car's speed as it coasts into the gas station on the other side of the valley?

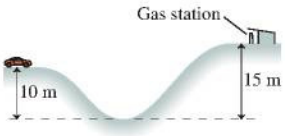


FIGURE EX10.11

12. | The maximum energy a bone can absorb without breaking is
BIO surprisingly small. Experimental data show that the leg bones of a healthy, 60 kg human can absorb about 200 J. From what maximum height could a 60 kg person jump and land rigidly upright on both feet without breaking his legs? Assume that all energy is absorbed by the leg bones in a rigid landing.

13. || A cannon tilted up at a 30° angle fires a cannon ball at 80 m/s from atop a 10-m-high fortress wall. What is the ball's impact speed on the ground below?

14. || In a hydroelectric dam, water falls 25 m and then spins a turbine to generate electricity.
    a. What is ΔU_G of 1.0 kg of water?
    b. Suppose the dam is 80% efficient at converting the water's potential energy to electrical energy. How many kilograms of water must pass through the turbines each second to generate 50 MW of electricity? This is a typical value for a small hydroelectric dam.

Section 10.3 Elastic Potential Energy

15. | How far must you stretch a spring with $k = 1000$ N/m to store 200 J of energy?

16. || A stretched spring stores 2.0 J of energy. How much energy will be stored if the spring is stretched three times as far?

17. | A student places her 500 g physics book on a frictionless table. She pushes the book against a spring, compressing the spring by 4.0 cm, then releases the book. What is the book's speed as it slides away? The spring constant is 1250 N/m.

18. | A block sliding along a horizontal frictionless surface with speed v collides with a spring and compresses it by 2.0 cm. What will be the compression if the same block collides with the spring at a speed of $2v$?

19. | A 10 kg runaway grocery cart runs into a spring with spring constant 250 N/m and compresses it by 60 cm. What was the speed of the cart just before it hit the spring?

20. || As a 15,000 kg jet plane lands on an aircraft carrier, its tail hook snags a cable to slow it down. The cable is attached to a spring with spring constant 60,000 N/m. If the spring stretches 30 m to stop the plane, what was the plane's landing speed?

21. || The elastic energy stored in your tendons can contribute up to
BIO 35% of your energy needs when running. Sports scientists find that (on average) the knee extensor tendons in sprinters stretch 41 mm while those of nonathletes stretch only 33 mm. The spring constant of the tendon is the same for both groups, 33 N/mm. What is the difference in maximum stored energy between the sprinters and the nonathletes?

22. || The spring in **FIGURE EX10.22a** is compressed by Δx. It launches the block across a frictionless surface with speed v_0. The two springs in **FIGURE EX10.22b** are identical to the spring of Figure EX10.22a. They are compressed by the same Δx and used to launch the same block. What is the block's speed now?

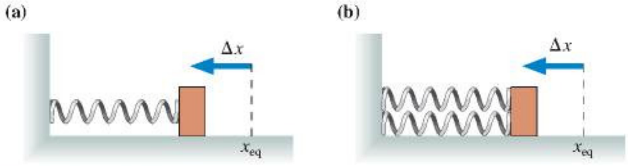


FIGURE EX10.22

23. ||| The spring in **FIGURE EX10.23a** is compressed by Δx. It launches the block across a frictionless surface with speed v_0. The two springs in **FIGURE EX10.23b** are identical to the spring of Figure EX10.23a. They are compressed the same *total* Δx and used to launch the same block. What is the block's speed now?

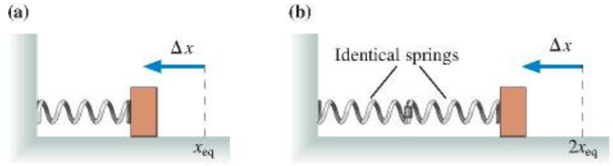


FIGURE EX10.23

Section 10.4 Conservation of Energy

Section 10.5 Energy Diagrams

24. || **FIGURE EX10.24** is the potential-energy diagram for a 20 g particle that is released from rest at $x = 1.0$ m.
    a. Will the particle move to the right or to the left?
    b. What is the particle's maximum speed? At what position does it have this speed?
    c. Where are the turning points of the motion?

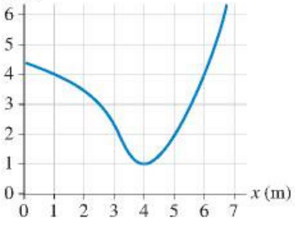


FIGURE EX10.24

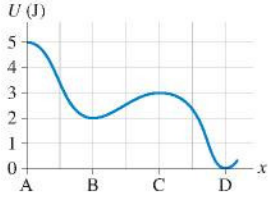


FIGURE EX10.25

25. || **FIGURE EX10.25** is the potential-energy diagram for a 500 g particle that is released from rest at A. What are the particle's speeds at B, C, and D?

26. || In **FIGURE EX10.26**, what is the maximum speed of a 2.0 g particle that oscillates between $x = 2.0$ mm and $x = 8.0$ mm?

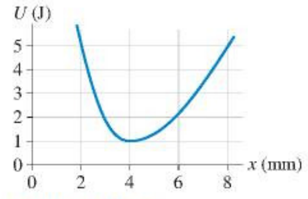


FIGURE EX10.26

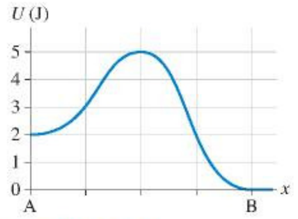


FIGURE EX10.27

27. | a. In **FIGURE EX10.27**, what minimum speed does a 100 g particle need at point A to reach point B?
  b. What minimum speed does a 100 g particle need at point B to reach point A?

28. || **FIGURE EX10.28** shows the potential energy of a 500 g particle as it moves along the x-axis. Suppose the particle's mechanical energy is 12 J.
  a. Where are the particle's turning points?
  b. What is the particle's speed when it is at $x = 4.0$ m?
  c. What is the particle's maximum speed? At what position or positions does this occur?
  d. Suppose the particle's energy is lowered to 4.0 J. Can the particle ever be at $x = 2.0$ m? At $x = 4.0$ m?

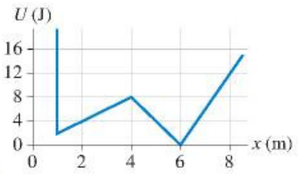


FIGURE EX10.28

29. || In **FIGURE EX10.28**, what is the maximum speed a 200 g particle could have at $x = 2.0$ m and never reach $x = 6.0$ m?

Section 10.6 Force and Potential Energy

30. || A system in which only one particle can move has the potential energy shown in **FIGURE EX10.30**. What is the x-component of the force on the particle at $x = 5$, 15, and 25 cm?

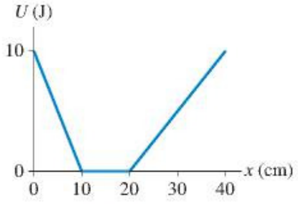


FIGURE EX10.30

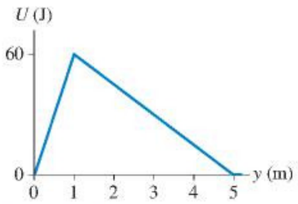


FIGURE EX10.31

31. || A system in which only one particle can move has the potential energy shown in **FIGURE EX10.31**. What is the y-component of the force on the particle at $y = 0.5$ m and 4 m?

32. || CALC A particle moving along the y-axis is in a system with potential energy $U = 4y^3$ J, where y is in m. What is the y-component of the force on the particle at $y = 0$ m, 1 m, and 2 m?

33. || CALC A particle moving along the x-axis is in a system with potential energy $U = 10/x$ J, where x is in m. What is the x-component of the force on the particle at $x = 2$ m, 5 m, and 8 m?

34. || **FIGURE EX10.34** shows the potential energy of a system in which a particle moves along the x-axis. Draw a graph of the force F_x as a function of position x.

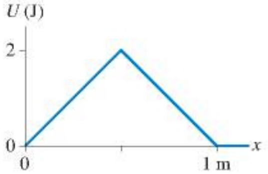


FIGURE EX10.34

Section 10.7 Conservative and Nonconservative Forces

35. | A particle moves from A to D in **FIGURE EX10.35** while experiencing force $\vec{F} = (6\hat{\imath} + 8\hat{\jmath})$ N. How much work does the force do if the particle follows path (a) ABD, (b) ACD, and (c) AD? Is this a conservative force? Explain.

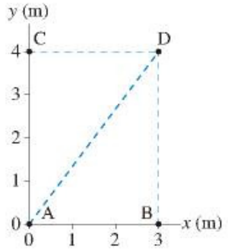


FIGURE EX10.35

36. | A force does work on a 50 g particle as the particle moves along the following straight paths in the xy-plane: 25 J from (0 m, 0 m) to (5 m, 0 m); 35 J from (0 m, 0 m) to (0 m, 5 m); –5 J from (5 m, 0 m) to (5 m, 5 m); –15 J from (0 m, 5 m) to (5 m, 5 m); and 20 J from (0 m, 0 m) to (5 m, 5 m).
  a. Is this a conservative force?
  b. If the zero of potential energy is at the origin, what is the potential energy at (5 m, 5 m)?

Section 10.8 The Energy Principle Revisited

37. | A system loses 400 J of potential energy. In the process, it does 400 J of work on the environment and the thermal energy increases by 100 J. Show this process on an energy bar chart.

38. | What is the final kinetic energy of the system for the process shown in **FIGURE EX10.38**?

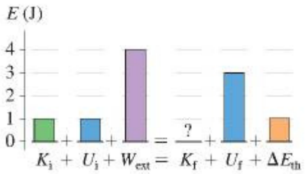


FIGURE EX10.38

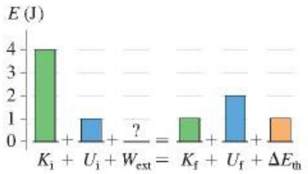


FIGURE EX10.39

39. | How much work is done by the environment in the process shown in **FIGURE EX10.39**? Is energy transferred from the environment to the system or from the system to the environment?

40. || A cable with 20.0 N of tension pulls straight up on a 1.50 kg block that is initially at rest. What is the block's speed after being lifted 2.00 m? Solve this problem using work and energy.

Problems

41. || A very slippery ice cube slides in a *vertical* plane around the inside of a smooth, 20-cm-diameter horizontal pipe. The ice cube's speed at the bottom of the circle is 3.0 m/s. What is the ice cube's speed at the top?

42. || A 50 g ice cube can slide up and down a frictionless 30° slope. At the bottom, a spring with spring constant 25 N/m is compressed 10 cm and used to launch the ice cube up the slope. How high does it go above its starting point?

43. || You have been hired to design a spring-launched roller coaster that will carry two passengers per car. The car goes up a 10-m-high hill, then descends 15 m to the track's lowest point. You've determined that the spring can be compressed a maximum of 2.0 m and that a loaded car will have a maximum mass of 400 kg. For safety reasons, the spring constant should be 10% larger than the minimum needed for the car to just make it over the top.
   a. What spring constant should you specify?
   b. What is the maximum speed of a 350 kg car if the spring is compressed the full amount?

44. || It's been a great day of new, frictionless snow. Julie starts at the top of the 60° slope shown in **FIGURE P10.44**. At the bottom, a circular arc carries her through a 90° turn, and she then launches off a 3.0-m-high ramp. How far horizontally is her touchdown point from the end of the ramp?

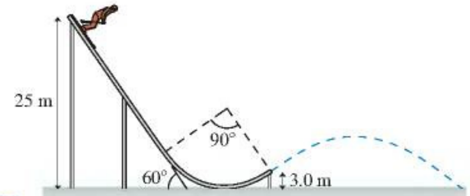


FIGURE P10.44

45. || A block of mass m slides down a frictionless track, then around the inside of a circular loop-the-loop of radius R. From what minimum height h must the block start to make it around without falling off? Give your answer as a multiple of R.

46. ||| A 1000 kg safe is 2.0 m above a heavy-duty spring when the rope holding the safe breaks. The safe hits the spring and compresses it 50 cm. What is the spring constant of the spring?

47. ||| You have a ball of unknown mass, a spring with spring constant 950 N/m, and a meter stick. You use various compressions of the spring to launch the ball vertically, then use the meter stick to measure the ball's maximum height above the launch point. Your data are as follows:

Compression (cm)	Height (cm)
2.0	32
3.0	65
4.0	115
5.0	189

Use an appropriate graph of the data to determine the ball's mass.

48. || Sam, whose mass is 75 kg, straps on his skis and starts down a 50-m-high, 20° frictionless slope. A strong headwind exerts a *horizontal* force of 200 N on him as he skies. Use work and energy to find Sam's speed at the bottom.

49. || A horizontal spring with spring constant 100 N/m is compressed 20 cm and used to launch a 2.5 kg box across a frictionless, horizontal surface. After the box travels some distance, the surface becomes rough. The coefficient of kinetic friction of the box on the surface is 0.15. Use work and energy to find how far the box slides across the rough surface before stopping.

50. || Truck brakes can fail if they get too hot. In some mountainous areas, ramps of loose gravel are constructed to stop runaway trucks that have lost their brakes. The combination of a slight upward slope and a large coefficient of rolling resistance as the truck tires sink into the gravel brings the truck safely to a halt. Suppose a gravel ramp slopes upward at 6.0° and the coefficient of rolling friction is 0.40. Use work and energy to find the length of a ramp that will stop a 15,000 kg truck that enters the ramp at 35 m/s (≈75 mph).

51. || A freight company uses a compressed spring to shoot 2.0 kg packages up a 1.0-m-high frictionless ramp into a truck, as **FIGURE P10.51** shows. The spring constant is 500 N/m and the spring is compressed 30 cm.
   a. What is the speed of the package when it reaches the truck?
   b. A careless worker spills his soda on the ramp. This creates a 50-cm-long sticky spot with a coefficient of kinetic friction 0.30. Will the next package make it into the truck?

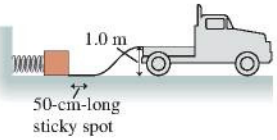


FIGURE P10.51

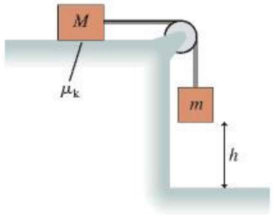


FIGURE P10.52

52. || Use work and energy to find an expression for the speed of the block in **FIGURE P10.52** just before it hits the floor if (a) the coefficient of kinetic friction for the block on the table is μ_k and (b) the table is frictionless.

53. || a. A 50 g ice cube can slide without friction up and down a 30° slope. The ice cube is pressed against a spring at the bottom of the slope, compressing the spring 10 cm. The spring constant is 25 N/m. When the ice cube is released, what total distance will it travel up the slope before reversing direction?
   b. The ice cube is replaced by a 50 g plastic cube whose coefficient of kinetic friction is 0.20. How far will the plastic cube travel up the slope? Use work and energy.

54. || The spring shown in **FIGURE P10.54** is compressed 50 cm and used to launch a 100 kg physics student. The track is frictionless until it starts up the incline. The student's coefficient of kinetic friction on the 30° incline is 0.15.
   a. What is the student's speed just after losing contact with the spring?
   b. How far up the incline does the student go?

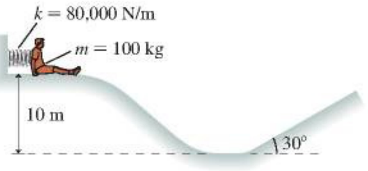


FIGURE P10.54

55. ||| CALC Protons and neutrons (together called *nucleons*) are held together in the nucleus of an atom by a force called the *strong force*. At very small separations, the strong force between two nucleons is larger than the repulsive electrical force between two protons—hence its name. But the strong force quickly weakens as the distance between the protons increases. A well-established model for the potential energy of two nucleons interacting via the strong force is

$$U = U_0\left[1 - e^{-x/x_0}\right]$$

where x is the distance between the centers of the two nucleons, x_0 is a constant having the value $x_0 = 2.0 \times 10^{-15}$ m, and $U_0 = 6.0 \times 10^{-11}$ J.

Quantum effects are essential for a proper understanding of nucleons, but let us innocently consider two neutrons as if they were small, hard, electrically neutral spheres of mass 1.67×10^{-27} kg and diameter 1.0×10^{-15} m. Suppose you hold two neutrons 5.0×10^{-15} m apart, measured between their centers, then release them. What is the speed of each neutron as they crash together? Keep in mind that *both* neutrons are moving.

56. || CALC A 2.6 kg block is attached to a horizontal rope that exerts a variable force $F_x = (20 - 5x)$ N, where x is in m. The coefficient of kinetic friction between the block and the floor is 0.25. Initially the block is at rest at $x = 0$ m. What is the block's speed when it has been pulled to $x = 4.0$ m?

57. || CALC A system has potential energy

$$U(x) = x + \sin\big((2 \text{ rad/m})x\big)$$

as a particle moves over the range $0 \text{ m} \le x \le \pi$ m.

a. Where are the equilibrium positions in this range?
b. For each, is it a point of stable or unstable equilibrium?

58. || CALC A particle that can move along the x-axis is part of a system with potential energy

$$U(x) = \frac{A}{x^2} - \frac{B}{x}$$

where A and B are positive constants.

a. Where are the particle's equilibrium positions?
b. For each, is it a point of stable or unstable equilibrium?

59. || A 100 g particle experiences the one-dimensional, conservative force F_x shown in **FIGURE P10.59**.

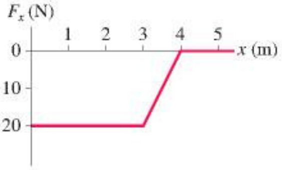


FIGURE P10.59

a. Let the zero of potential energy be at $x = 0$ m. What is the potential energy at $x = 1.0$, 2.0, 3.0, and 4.0 m?
Hint: Use the definition of potential energy and the geometric interpretation of work.
b. Suppose the particle is shot to the right from $x = 1.0$ m with a speed of 25 m/s. Where is its turning point?

60. || CALC A clever engineer designs a "sprong" that obeys the force law $F_x = -q(x - x_{\text{eq}})^3$, where x_{eq} is the equilibrium position of the end of the sprong and q is the sprong constant. For simplicity, we'll let $x_{\text{eq}} = 0$ m. Then $F_x = -qx^3$.

a. What are the units of q?
b. Find an expression for the potential energy of a stretched or compressed sprong.
c. A sprong-loaded toy gun shoots a 20 g plastic ball. What is the launch speed if the sprong constant is 40,000, with the units you found in part a, and the sprong is compressed 10 cm? Assume the barrel is frictionless.

61. || CALC The potential energy for a particle that can move along the x-axis is $U = Ax^2 + B\sin(\pi x/L)$, where A, B, and L are constants. What is the force on the particle at (a) $x = 0$, (b) $x = L/2$, and (c) $x = L$?

62. || CALC A particle that can move along the x-axis experiences an interaction force $F_x = (3x^2 - 5x)$ N, where x is in m. Find an expression for the system's potential energy.

63. || CALC An object moving in the xy-plane is subjected to the force $\vec{F} = (2xy\,\hat{\imath} + x^2\,\hat{\jmath})$ N, where x and y are in m.

a. The particle moves from the origin to the point with coordinates (a, b) by moving first along the x-axis to $(a, 0)$, then parallel to the y-axis. How much work does the force do?
b. The particle moves from the origin to the point with coordinates (a, b) by moving first along the y-axis to $(0, b)$, then parallel to the x-axis. How much work does the force do?
c. Is this a conservative force?

64. || CALC An object moving in the xy-plane is subjected to the force $\vec{F} = (2xy\,\hat{\imath} + 3y\,\hat{\jmath})$ N, where x and y are in m.

a. The particle moves from the origin to the point with coordinates (a, b) by moving first along the x-axis to $(a, 0)$, then parallel to the y-axis. How much work does the force do?
b. The particle moves from the origin to the point with coordinates (a, b) by moving first along the y-axis to $(0, b)$, then parallel to the x-axis. How much work does the force do?
c. Is this a conservative force?

65. Write a realistic problem for which the energy bar chart shown in **FIGURE P10.65** correctly shows the energy at the beginning and end of the problem.

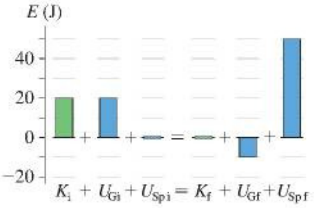


FIGURE P10.65

In Problems 66 through 68 you are given the equation used to solve a problem. For each of these, you are to

a. Write a realistic problem for which this is the correct equation.
b. Draw the before-and-after pictorial representation.
c. Finish the solution of the problem.

66. $\frac{1}{2}(1500 \text{ kg})(5.0 \text{ m/s})^2 + (1500 \text{ kg})(9.80 \text{ m/s}^2)(10 \text{ m})$
$= \frac{1}{2}(1500 \text{ kg})v_i^2 + (1500 \text{ kg})(9.80 \text{ m/s}^2)(0 \text{ m})$

67. $\frac{1}{2}(0.20 \text{ kg})(2.0 \text{ m/s})^2 + \frac{1}{2}k(0 \text{ m})^2$
$= \frac{1}{2}(0.20 \text{ kg})(0 \text{ m/s})^2 + \frac{1}{2}k(-0.15 \text{ m})^2$

68. $\frac{1}{2}(0.50 \text{ kg})v_f^2 + (0.50 \text{ kg})(9.80 \text{ m/s}^2)(0 \text{ m})$
$+ \frac{1}{2}(400 \text{ N/m})(0 \text{ m})^2 = \frac{1}{2}(0.50 \text{ kg})(0 \text{ m/s})^2$
$+ (0.50 \text{ kg})(9.80 \text{ m/s}^2)((-0.10 \text{ m}) \sin 30°)$
$+ \frac{1}{2}(400 \text{ N/m})(-0.10 \text{ m})^2$

Challenge Problems

69. ||| A pendulum is formed from a small ball of mass m on a string of length L. As **FIGURE CP10.69** shows, a peg is height $h = L/3$ above the pendulum's lowest point. From what minimum angle θ must the pendulum be released in order for the ball to go over the top of the peg without the string going slack?

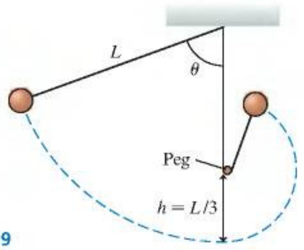


FIGURE CP10.69

70. ||| In a physics lab experiment, a compressed spring launches a 20 g metal ball at a 30° angle. Compressing the spring 20 cm causes the ball to hit the floor 1.5 m below the point at which it leaves the spring after traveling 5.0 m horizontally. What is the spring constant?

71. ||| It's your birthday, and to celebrate you're going to make your first bungee jump. You stand on a bridge 100 m above a raging river and attach a 30-m-long bungee cord to your harness. A bungee cord, for practical purposes, is just a long spring, and this cord has a spring constant of 40 N/m. Assume that your mass is 80 kg. After a long hesitation, you dive off the bridge. How far are you above the water when the cord reaches its maximum elongation?

72. ||| CALC A 10 kg box slides 4.0 m down the frictionless ramp shown in **FIGURE CP10.72**, then collides with a spring whose spring constant is 250 N/m.
    a. What is the maximum compression of the spring?
    b. At what compression of the spring does the box have its maximum speed?

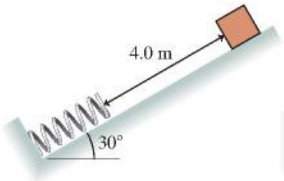


FIGURE CP10.72

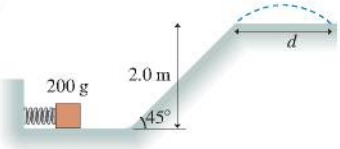


FIGURE CP10.73

73. ||| The spring in **FIGURE CP10.73** has a spring constant of 1000 N/m. It is compressed 15 cm, then launches a 200 g block. The horizontal surface is frictionless, but the block's coefficient of kinetic friction on the incline is 0.20. What distance d does the block sail through the air?

74. ||| A sled starts from rest at the top of the frictionless, hemispherical, snow-covered hill shown in **FIGURE CP10.74**.
    a. Find an expression for the sled's speed when it is at angle ϕ.
    b. Use Newton's laws to find the maximum speed the sled can have at angle ϕ without leaving the surface.
    c. At what angle ϕ_{max} does the sled "fly off" the hill?

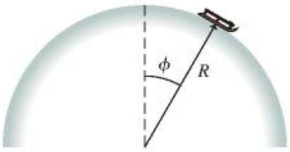


FIGURE CP10.74